THE PRINCIPLES OF ARITHMETICKE,

METHODICALLY DIGESTED, and by short and familiar examples illustrated and declared:

Together with the Art of Allegation:

First written in Latine by BERNARD SAVILLAGRE, *and now Englished by* WILLIAM BEDWELL.

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THE PREFACE TO THE READER.

ALthough I am not ignorant, courteous Reader, that there are in our language already very many & ſundry bookes imprinted of this argument: and that therefore to write ought in this kind, were but loſt labour, or more then needeth: Yet for that, as the learned do know, moſt of them are too long and tedious; and ſo withall weariſome vnto the teacher: and many of them are meere *praxeis*, without any generall rules or grounds of Art, and by that meanes are not ſo good for the learner; I haue often wiſhed, that ſome other which for Art and Methode were more fit both for the one and the other, might yet by ſome Louer of theſe ſtudies be publiſhed. This hauing long in vaine expected, I thought it my part to do

that, which I would willingly haue impoſed vpon others. And finding none, in mine opinion, to haue in all points excelled *Bernard Saligwacke*, we haue made choiſe of him aboue others, whom, for the benefite of ſome priuate friends, we haue aboue twenty yeares ſince tranſlated into our Engliſh tongue. Now whereas he, as the learned do know, hath not onely orderly digeſted the rules of this Art, which for the moſt part *Ramus* his maiſter before him had done: but hath alſo moſt learnedly and curiouſly demonſtrated the ſame, which no man before him, for ought I know, ſince *Euclides* time hath attempted: Theſe demonſtrations of his, being beyond the reach and capacity of ſuch as are not well grounded in other more generall arts and ſciences, we haue wholly omitted. And haue onely retained the grounds and principles of the Art, that is the rules and axiomes of the ſame, without any alteration at all, with the examples, and whatſoeuer elſe might ſerue for the illuſtration and declaration thereof. Yet if we ſhall hereafter vnderſtand,

ſtand, that they ſhall be alſo deſired, we haue them likewiſe in a readineſſe: and thoſe ſo digeſted and ordered, according to an inuention of ours, that with farre more facilitie they may be conceiued, then ordinarily ſuch like demonſtrations of the Mathematicians, as they are by them layd downe, poſſibly may, euen of thoſe that are moſt acute and beſt of apprehenſion. Of our owne we haue added very little: For that indeed in him little or nothing, in our opinion, is wanting that may be thought greatly neceſſary. One thing we are here to admoniſh the Reader of; That whereas any greater number, written I meane in more figures then foure or fiue, do occurre or ſhall be met withall, there for the moſt part you ſhall find them diſtinguiſhed into ternaries or three degrees, according to the preſcript of the 12. verſe of the firſt Chapter of the firſt booke, that by that meanes they may be the eaſilier pronounced without ſtop or ſtay of the Reader. For this we haue the authority and warrant of the beſt and moſt famous Mathematicians of our

time, I meane of *Lansbergius*, *Hadrianus Romanus*, *Franciſcus Vieta*, and others. If mine occaſions and buſineſſes had not bene ſuch as might not be neglected, I would to theſe haue alſo adioyned the *Algebra* of the ſame Author, being of all, amongſt very many that are written of this argument, in our iudgement, the moſt eaſie and fitteſt for the capacity of thoſe for whom this labour of ours was firſt taken in hand. This therefore, if our leyſure may better ſerue, expect with the next edition. Farewell, this 20. of Aprill. 1616.

Thine in the Lord,

WILLIAM BEDVVEL.

THE FIRST BOOKE OF ARITHMETICKE.

CHAP. I.

Of the Definition and Diuiſion of Arithmeticke: and of the Notation of numbers.

1 ARithmeticke is the art of numbring.

By numbers we vnderſtand, not onely numbers of many: but alſo an vnity: yea and the parts of an vnity; or any other kind of parts. For by theſe we do alſo number: and a number is that whereby any thing is numbred, as a meaſure is that whereby any thing is meaſured.

2 The parts of Arithmeticke are two: the firſt part is that which declareth the Notation and

and Numeration of numbers.

The parts of Arithmeticke distinguished in nature, cannot fitly be expressed in words.

3 Notation is the exposition of the writing of numbers: his parts are two.

Notation (falsly by some called Numeration) teacheth the manner of writing of numbers; and the expressing of numbers written.

4 The first part of Notation doth declare the value of the figures wherewith numbers are written. The figures wherewith we write numbers are these ten: 1,2,3,4,5,6,7,8,9,0.

That those figures were found out by the Arabians many arguments do euince. The Hebrewes & Greeks haue in this case vsed all the letters of their alphabets: the Romanes onely these seuen, I V X L C D M. But now for these foure, or fiue hundred yeares, all nations with one consent haue generally vsed those aboue written.

5 These figures are considered either by themselues, or one with another. The first nine are significant.

Figures are considered by themselues, when as they stand alone, or are so considered as if they stood alone. The first nine, I say, do signifie somewhat. So 1, doth signifie one: 2, two: 3, three: 4, foure: 5, fiue: 6, sixe: 7, seauen: 8, eight: 9, nine.

6 The

6 The tenth by himselfe signifieth nought. Therefore of the vse it is called a Nought: or of the forme, a Circle: or by the generall name, a Cypher.

7 Figures considered betweene themselues, are distinguished by Degrees, and Periods. A Degree is a place by which the value of the figure is esteemed.

Figures are considered betwéene themselues being in composition, that is, being ioyned one with another. And the succession of places increaseth the value of the figures. Degrées are counted and estéemed from the right hand toward the left.

8 A Degree is twofold, a Former, or Follower. The Former degree is that in which ŷ value of the figure is the same that it was by it selfe. The Following degree is that in which the value of the figure is manifold vnto the former; and it is the Second or Third. The second is tenfold to the former, the third an hundred fold vnto the same.

The figure of ŷ first degrée hath none other value then was assigned it by the 4 verse, but in ŷ following degrées it getteth another value. So this figure 1, in the first degrée is but one, in the degrées following it also signifieth one, but in the second degrée one tenne; in the third

third one hundred: In like manner of the rest. Here therefore euery signifying figure is armed with a double power and value: The first naturall and proper to the figure, the other borrowed, and giuen him by reason of his place. A Cypher therefore, which by himselfe signifieth nought, by the 6 verse, being also ioyned with others, still signifyeth nought, but onely by filling of a place increaseth the value of the degrées following.

9 Thus far of the degrees. A period is a certaine company of degrees, and it is first either perfect or imperfect. A perfect period is that which consisteth of a perfect ternary of degrees. As in these periods 100. 100000. 100000000. an hundred, an hundred thousand, an hundred thousand thousands: here alwaies are perfectly three degrées, that is to say, in the first example once, in the second twise, in the third thrise: wherefore such like periods are perfect.

10 An imperfect period is that which consisteth of an vnperfect ternary of degrees.

I call that an imperfect period which is lesse then a perfect ternary equally manifold: as in the periods 10. 1000. ten, a thousand. Here ỳ ternary of the first period is but one, which a perfect period exceedeth by one degrée. The ternary of the second is double, which a perfect

perfect period exceedeth by two degrées. Therefore such like periods are vnperfect.

11 Againe, a period is either single or compound. A single period is that which consisteth of one ternary of degrees onely.

As 10. 100. here the ternary of degrées is but one in each number, that is, in the first example, imperfect: in the second perfect, therefore such like periods are single or simple.

12 A compounded period is that which consisteth of more then one ternary of degrees.

As in these examples 1000. 10000. 10000000. here the ternary of degrées in the two first examples is double, in the third triple. Therefore such like periods are compound. Simple periods are esteemed and counted by the 8 verse: Compounded periods by these speciall rules following.

1 A compounded period is to be distinguished into his perfect simple periods after the order of the places.

2 One single period following exceedeth the next former one thousand fold. Therefore

3 So many times thousand is to be put into the period following, as there be periods before him.

This distinction some do make by strikes drawne betweene the figures: other some by pricks

prickes or points, either placed aboue or beneath the figures. The learned Mathematicians at this time do vse the grammarians comma; which therefore I do thinke most fit, and to be vsed alwaies where any greater compound period is to be written; and that for the more speede of the reader.

Therefore, hauing a number written with these figures 78500642145. I distinguish it into his perfect single periods, beginning at ỹ right hand, thus 78,500,642,145. The same I thus pronounce, seauenty eight thousand thousand thousands: fiue hundred thousand thousands: fiue hundred forty two thousands: one hundred fortie and fiue.

You may for brefity sake call the third period Millions, and the fourth, Milliards.

CHAP. II.

Of the second part of Notation.

1 THE second part of Notation is that which declareth the Bounds and Kinds of numbers.

2 The Bounds are they which stand about the number, and do enclose it.

As in bignelles, so in numbers there are certains

taine bounds, although after another manner: for there betweene the bound & the thing bounded is some difference; here is none.

3 The Bounds are either Numerators or Denominators. Those do number the enclosed number, these do name it. And when they are written together, the Numerators are placed aboue the Denominators.

As here $\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{2}{3}$, one vnitie, two vnities; one second, two thirds. The vpper bounds are commonly called Numbers or Numerators; the neathermost, Names or Denominators. These do name into how many parts one thing is deuided: those do number the parts of such a diuision. The truth of this distinctiõ is most apparent in parts: but in whole numbers it is not so easily perceiued; for that true whole numbers commonly are both written and pronounced without their denominators for breuity sake. But accustomed and vsuall breuity doth not take away the truth and nature of things.

4 The kinds of numbers, I do call the numbers enclosed with bounds. Such like numbers are either First or Secondary. First are those whose knowledge is first, & dependeth of none other. And they are the True whole numbers, or True parts. True whole numbers are those

whose

whose denominator is alwaies an vnity: therefore they are commonly written in one row.

As thus $\frac{3}{1}, \frac{2}{1}, \frac{1}{1}$, one vnitie, two vnities, three vnities: For indeede when we say one, two, three, we vnderstand one, two, three vnities. But because that ỹ name of true whole numbers is alwaies an vnitie; therefore we vse commonly (I say commonly, for that in mingled proportioned numeration it is otherwise,) both to pronounce and write their numerators onely: for this course is shorter and more easie.

5 True whole numbers, are either numbers of one, or of many. One, or an vnitie, is a whole number whose numerator is 1: and it is the beginning of a multitude. A number of many is a whole number, whose numerator is greater then 1. And a multitude is a company of vnities.

An vnity I say is that whose numerator is 1, as $\frac{1}{1}$: or else whose numerator is equall to the denominator, which (as we shall afterward in Reduction vnderstand) is all one: so $\frac{1}{1}$, or $\frac{4}{4}$, are 1. A number of many is that whose numerator is greater then 1, such are $\frac{2}{1}, \frac{3}{1}, \frac{14}{7}$, $\frac{1000}{3}$. Therefore

6 The least of whole numbers is one, the greatest cannot be giuen.

7 True parts are those whose denomina-

to

tor is greater then their numerator, and are alwaies written in two rowes, one aboue another, with a line drawne betweene them.

As here $\frac{1}{2}, \frac{35}{36}$, that is one halfe, and thirtie fiue thirtie sixes: And generally, as I said, as the denominator of true parts is an vnitie, so here it is alwaies some number of many.

8 A part of a lesser denominator is greater then a part of a greater denominator.

So $\frac{1}{2}$, is greater then $\frac{1}{3}$: and $\frac{1}{3}$, is greater then $\frac{1}{4}$. Therefore

9 The greatest of single parts is $\frac{1}{2}$, the least cannot be giuen.

I call those single parts whose numerator is 1. Which here is the greatest, as it was the least of whole numbers, verse 6. The least cannot be told, as neither the greatest of whole numbers, by the same verse.

10 Thus far of first numbers. Secondary numbers are those whose knowledge is secondary. And they are simple or compounded.

By secondary knowledge I vnderstand such a knowledge as cannot be gotten without the knowledge of some one thing vpon which it doth depend, do go before it. So what secondary whole numbers, and secondary parts do meane, cannot be vnderstood except one first do know what the first whole numbers and

first parts do meane.

11 Simple secondary numbers are written in two rowes onely: and they are either whole numbers or parts. Whole secondary numbers are written without a line betweene them.

As in $\frac{2}{3}\,\frac{4}{7}$, $\frac{3}{4}\,\frac{5}{8}\,\frac{7}{9}$. Two thirds of foure seauenths: Three fourths of fiue eights of seauen ninths. In the first example $\frac{4}{7}$, are whole ones onely: In the second $\frac{5}{8}$, are whole ones in respect of the former; and parts in respect of the followers.

12 Parts are written in their rankes with a line betweene. If their ouermost ranke be equall to the neathermost, it signifieth that the parts are equall to one vnitie: If it be greater, that they be greater then an vnity.

As in $\frac{2}{2}$, and $\frac{3}{2}$: there the parts be equall to an vnitie, here they be greater then an vnitie.

13 Compound secondary numbers are those which are written in more rankes then two.

So $2\,\frac{1}{2}$, is a compound secondary number. Thus farre of Notation; Numeration followeth.

CHAP.

CHAP. III.

Of Numeration in generall: And of equalled Numeration in ſpeciall.

1 NVmeration is that which of two numbers giuen doth find out the third lawfully.

Hereupon it is manifeſt, If more then two numbers be giuen to be numbred, that two are onely to be numbred: For Numeration doth find the third onely of two aſſigned. Therefore if 3, 4, and 7, were to be added together: firſt 3 are to be added to 4, and then the ſumme of them to 7.

Lawfully] Numeration may not find out the third any kind of way, but it muſt do it lawfully: If therefore the rules of Numeration do not ſuffer the finding of the third, as when 2, are to be taken out of 2, or 3, out of 3, Numeration ſhall find out no third number.

2 Having two numbers giuen, written in two figures, they are wholly to be numbred: In many, by parts.

So we ſay 3, and 4, are 7: and three times 4, are 12. But if 52, were to be added to 64, here firſt we adde 4, to 2, then 6, to 5. If 7, were to be multiplyed by 36. Firſt we multiply 7, by 6, then ỹ ſame by 3: For it is all one to mul-

tiply by the whole, and by the parts. But in the Numeration by parts it is somewhat more easie: Because we do more easily number small numbers then great.

3 In this kind of numeration two things are to be marked: the first is, that the figures in numbring are to be considered as if they stood alone.

So in the addition of 47 to 32, 2 and 7 being added, the figures remaining are not to be called 40, and 30, but are to be considered as 3, and 4: for so it is greater easinesse.

4 The other is: If the figure set downe be to be blotted out afterward, it is to be kept in mind, and to be numbred with the figures following.

In addition of 56, to 38, first we say 8 and 6 are 14, but then adding 5 to 3, the 1 afore written were to be blotted out: therefore I keepe it in mind: for so the worke shall be shorter.

5 We make a double distinction of Numeration. First, Numeration is equalled or proportioned. Equalled Numeration is that to which equality prescribeth a rule. And it is Addition or Subduction. Addition beginneth at the right hand, and addeth the numbers giuen, that it may find the summe equall to them.

So 10, the summe of 3, and 7, is equall, to the

the ſame 3 and 7: for we make the whole equall to all his parts ioyntly taken. The proper ſigne of Addition ſignifying more, is this +: ſo we ſay 3 + 7 are 10, that is, 3 more by 7, or 3 and 7 are 10.

6 The properties of Addition are two: the firſt is thus: If equall numbers be added to equall, the ſummes ſhall be equall.

So if you adde 4, and 4, two equall numbers to 7, and 7, two equall numbers, the ſummes 11, and 11, ſhall be alſo equall.

7 The ſecond is thus: If you adde equall numbers to vnequall, the ſumme likewiſe ſhall be vnequall to the vnequall added.

If to the aſſigned 5, and 6, you adde 4, and 4, the ſummes 9, and 10, ſhall be likewiſe vnequall to the added 5, and 6: that is, by how much 5, is leſſer then 6, by ſo much 9, ſhall be leſſer then 10.

8 Subduction beginneth at the left hand, and taketh the one of the aſſigned numbers out of the other, that it may find the remaine with that taken out, equall to that from which the Subduction was made.

So 7, being taken out of 10, there remaineth 3, which 3, with 7, ſhall be equall to the ſame 10. for we make all the parts ioyntly taken, equall to the whole. The proper ſigne of Subduction

tiply by the whole, and by the parts. But in the Numeration by parts it is somewhat more easie: Because we do more easily number small numbers then great.

3 In this kind of numeration two things are to be marked: the first is, that the figures in numbring are to be considered as if they stood alone.

So in the addition of 47 to 32, 2 and 7 being added, the figures remaining are not to be called 40, and 30, but are to be considered as 3, and 4: for so it is greater easinesse.

4 The other is: If the figure set downe be to be blotted out afterward, it is to be kept in mind, and to be numbred with the figures following.

In addition of 56, to 28, first we say 8 and 6 are 14, but then adding 5 to 3, the 1 afore-written were to be blotted out: therefore I keepe it in mind; for so the worke shall be shorter.

5 We make a double distinction of Numeration. First, Numeration is equalled or proportioned. Equalled Numeration is that to which equality prescribeth a rule. And it is Addition or Subduction. Addition beginneth at the right hand, and addeth the numbers giuen, that it may find the summe equall to them.

So 10, the summe of 3, and 7, is equall, to the

the ſame 3 and 7: for we make the whole equall to all his parts ioyntly taken. The proper ſigne of Addition ſignifying more, is this +: ſo we ſay 3 + 7 are 10, that is, 3 more by 7, or 3 and 7 are 10.

6 The properties of Addition are two: the firſt is thus: If equall numbers be added to equall, the ſummes ſhall be equall.

So if you adde 4, and 4, two equall numbers to 7, and 7, two equall numbers, the ſummes 11, and 11, ſhall be alſo equall.

7 The ſecond is thus: If you adde equall numbers to vnequall, the ſumme likewiſe ſhall be vnequall to the vnequall added.

If to the aſſigned 5, and 6, you adde 4, and 4, the ſummes 9, and 10, ſhall be likewiſe vnequall to the added 5, and 6: that is, by how much 5, is leſſer then 6, by ſo much 9, ſhall be leſſer then 10.

8 Subduction beginneth at the left hand, and taketh the one of the aſſigned numbers out of the other, that it may find the remaine with that taken out, equall to that from which the Subduction was made.

So 7, being taken out of 10, there remaineth 3, which 3, with 7, ſhall be equall to the ſame 10. for we make all the parts ioyntly taken, equall to the whole. The proper ſigne of Sub-

duction signifying lesse, is thus——. So we say 10——7, are 3, that is, 10, lesse by 7: Or 7, subtracted out of 10, the remaine shall be 3: Or 10——3 are 7, that is, 10, lesse by 3: Or 3, taken out of 10, there shall remaine 7.

9 The properties of Subduction are two: The first is thus: If you take equal numbers out of equall, the remaines shall be equall.

So 3, and 3, subducted out of 9, and 9, the remaines 6, and 6, shall be equall.

10 The second is thus: If you take equall numbers out of vnequall: or vnequall out of equall, the remaines shall be likewise vnequall directly or contrariwise to the vnequall giuen.

So 4, and 4, being subducted out of 7, and 6, the remaines 3, and 2, shall be likewise vnequall to the assigned 7, and 6. Or thus, by how much 7 doth exceede 6: by so much 3, doth exceede 2. Againe, 3, and 5, subducted out of 9, and 9, the remaines 6, and 4, shall be likewise vnequall to ỹ assigned 5, and 3: Or thus, by how much 3, is lesser then 5, by so much 6, is greater then 4.

CHAP.

CHAP. IIII.

Of proportioned Numeration, and especially of Multiplication.

1 PRoportioned Numeration is that, to which proportion preſcribeth a rule: and it is Multiplication or Diuiſion.

2 Multiplication beginneth at the right hand, and multiplieth the aſſigned betweene themſelues, that it may make a number which is to one of the aſſigned as the other is to an vnitie.

So when we multiply 3 by 4, we ſeeke a number which is to 3. as 4, is to 1: Or which is to 4, as 3, is to 1. Here the number found is properly called the Fact or Product.

3 If two numbers multiply one the other any way; their products ſhall be equall.

Whether you multiply 2, by 4. or 4, by 2, you ſhall make equall numbers for their products: For the way from Cambridge to London, is the way from London to Cambridge.

4 If one number do multiply many, and their ſumme, the product of the ſumme ſhall be equall to the products of the others.

For it is all one to multiply by the whole and by the parts. Therefore the products of

4, by 3, 5, and 7, shall be equall to the product of their summe 15, by the same 4.

5 Parts shall be equall betweene themselues, when the products of their Numerators by their each other Denominators, shall be equall betweene themselues.

This is a kind of definition of the equality of parts, drawne out of the crosse Multiplication of their bounds. Therefore in $\frac{2}{4}$, and $\frac{3}{6}$, if the product of 4, by 3, be equall to the product of 2, by 6, the parts $\frac{2}{4}$, and $\frac{3}{6}$, shall be equall betwæne themselues.

6 If three numbers vnequall betweene themselues be giuen, the product of the summe of the differences, by the middle number, shall be equall to the products of the differences by each other extreame.

The thræe numbers vnequall betwæne thẽselues let them be 16, 11, and 9. and then let the differences of the meane 11, from the extreames 16, and 9, be 5, and 2, and let the summe of them be 7.

16 2

11 7

9 5

The product of 7, by 11 that is 77 shall be equall to the products of 2, by 16, and 5, by 9, that is 32, and 45. This proposition is the sole

ſole and onely groundworke of *Archimedes* his Allegation.

Here I thinke it not amiſſe to ſhew ſome ſpeciall rule of Multiplying of ſingle figures. Amongſt many, I vſe this as beſt.

Hauing two ſingle figures giuen, whoſe ſumme is more then ten; the product of their differences from ten, added to the product of the compounds digit made by ten, ſhall be the product of the numbers giuen.

Let the numbers giuen, be 8, and 7, whoſe ſumme 15, is more then ten: the digit of which compound number 5, I multiply by 10 and ỹ product is 50. Now the differences of 8, from 10, and 7, from 10 are 2, and 3, which I likewiſe multiply betweene themſelues, and I make the product 6: Which I adde to the former product, 50, and I find the ſumme 56, for the product of 8, by 7, which was deſired.

This is true onely when the two numbers aſſigned being added do make more then ten: and indeede there is no great neede of any rule in ſuch as are ſmaller. Another example. Let 5, and 7, be giuen: Here 5 + 7, are 12: then 2, by 10, are 20: and 10 — 5, are 5, and 10 — 7, are 3: then 5, by 3, make 15, laſtly 20 + 15, are 35, the product of 7, by 5, which was deſired.

CHAP.

CHAP. V.

Of Diuision.

1 Diuision beginneth at the left hand, and diuideth the one of the numbers giuen by the other, that the quotient may be had, which is vnto the Diuidend as an vnity is vnto the Diuisor.

So when we deuide 12, by 4, we seeke a quotient, which is vnto 12, as 1, is vnto 4. The properties of Diuision do follow.

2 If the Diuisor be equall vnto the Diuidend, it shall measure the Diuidend by 1.

So 4 doth measure 4, by 1: and 6, doth measure 6, by 1. Generally that number is said to measure an assigned number, which being subducted out of it as oft as it may, doth leaue nought. Whereupon it followeth that a greater number may not measure a lesser: because you cannot subduct the greater from the lesser. And the quotient found is some one whole number. Lastly, to measure is to deuide; but not contrariwise.

3 If the Diuisor be greater then the Diuidend, the quotient shall yeeld true parts, whose Numerator is the Diuidend; the Denominator the Diuisor.

So when we deuide 5, by 12, the quotient is $\frac{5}{12}$.

4 If

4 If the Diuiſor be leſſer then the Diuidend, it ſhall be either a part of the Diuidend, or parts of the ſame.

Here I oppoſe a part to parts, as before at ỳ 5 verſe of the ij Chap. I oppoſed one to many.

5 If the Diuiſor be a part of the Diuidend, the Numerator of ſuch like parts is an vnitie, the Denominator ſhall be the quotient it ſelfe.

So if 4, which is leſſe then 20 ſhall meaſure the ſame 20, by 5, then I ſay that 4, ſhall be $\frac{1}{5}$ of 20.

6 If the Diuiſor do make parts of the Diuidend, the Numerator of ſuch like parts ſhall be the Diuiſor, their Denominator the Diuidend it ſelfe.

The Diuiſor maketh parts of the Diuidend, which doth not meaſure a Diuidend greater then it ſelfe, but being taken out of the Diuidend, ſo often as may be, doth leaue ſome thing leſſe thẽ himſelfe. As 7, ſubducted out of 20, ſo oft as may be, leaueth 6, which remaine is leſſe then 7: therefore the Diuiſor 7, ſhall be $\frac{7}{20}$ of 20.

7 A number to be deuided of an aſſigned by the third, is deuided of the third by the aſſigned.

If 12, may be deuided of 4, by 3, it ſhall alſo be deuided of 3, by 4.

8 If

8 If one number deuide many and the ſumme of them, the quotient of the ſumme ſhall be equall to the quotients of the others.

It is all one to deuide by the whole and by the parts. Therefore the quotients, of 6, 12, and 16, by 2, ſhall be equall to the quotient of their ſumme 34, by the ſame 2. Therefore

9 If a number do meaſure many numbers, it ſhall meaſure the ſumme of them alſo.

Other wiſe the quotient of the ſumme ſhould be vnequall to the quotients of the particulars, againſt the 8 verſe. And

10 If a number do meaſure an aſſigned, and one part thereof, it ſhall alſo meaſure the other.

The cauſe is the ſame with the former.

11 If a number do meaſure the meaſure of the aſſigned, it ſhall alſo meaſure the aſſigned.

For the meaſurer of the meaſure of the aſſigned, repeated ſo oft as may be, is the meaſure of the aſſigned.

12 If a number do meaſure a number, the meaſurer ſhal be the greateſt common meaſure of them both, and the deuided ſhall be the leaſt to be deuided of them both.

The common meaſure is a number which doth meaſure the aſſigned, and that is the greateſt if no other greater common meaſurer may

may be giuen. In like manner the common Diuidend is a number which the assigned do measure, and that is the least, if no lesser common Diuidend may be giuen. Now suppose that the assigned 4. doth measure the assigned 12. I say that 4 is the greatest common measure of 4 and 12. For 4, doth measure himselfe by the 2 verse, and it measureth 12, by the grant. And no greater can measure them both; because a greater number then 4, cannot measure 4, by the commentary of the 2 verse. Againe, I say that 12, is the least common Diuidend of 4, and 12: for 12 is measured of it selfe, by the 2 verse, and of 4, by the grant: and no lesser can measure thē both, for then 12 should measure a lesse then himselfe contrary to the commentary of the 2 verse.

13 If two numbers assigned do measure another, the least Diuidend of them shal measure the same.

Let two numbers, 4 and 6, measure 24, and the least Diuidend of them let it be 12, I say 12, shall measure 24.

14 If foure numbers be proportionall, the same also shall be proportionall crosse wayes and backward.

The bounds of proportion are crossed or thwarted, when one of the formers is made a follower;

follower; they are inuerted (or considered backward) when both of the antecedents or formers are made followers or consequents. As 2 are to 3: so are 4 to 6. Therefore crosse waies, as 2 are to 4: so are 3 to 6: Or as 4 are to 2: so are 6 to 3. Againe, as 2 are to 3: so are 4 to 6. Therefore backward, as 3 are to 2: so are 6 to 4: Or as 6 are to 4: so are 3 to 2.

CHAP. VI.

Of the properties of numbers, taken out of Multiplication and Diuision.

1 IF a number be made of two numbers assigned, one of the assigned shall deuide it by the other: and contrariwise.

If 12 be made of 4. by 3. then 4 shall deuide 12 by 3: and 3 shall deuide the same 12 by 4. Againe, if 4 do deuide 12, by 3: I say 12 is made by the multiplication of 4, by 3.

2 If of three numbers giuen, the first shall deuide either of the followers, and the quotient do multiplie the other, the product shall be the fourth proportionall vnto the three giuen.

Let the three numbers giuen be 3, 15, and 12. And then let 3 deuide 15, by 5: and the product of 5, by 12, let it be 60. I say that 60 the product

product of 5, by 12, shall be the fourth proportionall. And therefore as 5, is to 15: so is 12, to 60.

3 If of three numbers giuen the second shall deuide one of the other, and the quotient do multiply the remainer, as the first is vnto the second, so shall the product be vnto the third.

Let the three numbers giuen be 8, 4, and 6: and the quotient of 8, by 4, let it be 2: and the product of 2, by 6, let it be 12: I say as 8 is to 4: so 12 is to 6.

4 If a number do multiply many numbers, the products shall be proportionall vnto the multiplied.

Let 2 multiply 4, and 3, and make 8 and 6: I say as 4, is to 3, so is 8, to 6.

5 If foure numbers be proportionall, the product of the outmost shall be equall to the product of the middlemost.

This is commonly called the Golden rule: and certainly the Mathematicall Sciences do daily reape from hence more precious fruites then gold in value. As 12 is to 4: so is 6 to 2: I say therefore that the product of 12, by 2, is equall to the product of 6, by 4.

Therefore

6 If of three numbers assigned the second do multiplie the third, and the first shall deuide

the

the product of them, the quotient shall be the fourth proportionall vnto the three giuen.

This is manifest by the 1 verse of this Chapter. And

7 If the bounds of parts be proportionall, the parts shall be equall betweene themselues.

This is manifest by the 5 verse of this Chapter.

8 If the product of the two middle numbers shall be equall to the product of the extreames, the foure assigned shall be proportionall.

This is the contrarie of the Golden rule. Let the foure numbers assigned be 12, 4, 6, 2: I say if the product of 4, by 6 be equall to the product of 12, by 2, that the foure assigned are proportionall: therefore as 12 are to 4, so are 6 to 2. Therefore

9 If a number be made of two numbers assigned, his Diuisor and Quotient shall be the meane proportionals vnto the assigned.

This is manifest by the first verse of this Chapter. And

10 If of three numbers assigned the first shall multiply the third, and the second do deuide the product; as the first is vnto the second, so shall the quotient be vnto the third.

The truth of this appeareth also by the 1 verse of this Chapter.

11 If

11 If foure proportionall numbers ſhall multiply ſo many proportionals, the products alſo ſhall be proportionall betweene themſelues.

As 3 is to 5, ſo let 15 be to 25: And as 6 is to 12, ſo let 7 be to 14. Againe, let the products of 3 by 6, of 5 by 12; of 15 by 7: of 25 by 14, be 18, 60, 105, and 350: I ſay as 18 is to 60, ſo is 105 to 350. The example is thus.

3	5	15	25
6	12	7	14
18	60	105	350

CHAP. VII.

Of the ſecond Diuiſion of Numeration: and of Addition and Subduction of true whole numbers.

1 THus farre we haue ſpoken of equalled and proportioned Numeration: the other diſtinction of Numeration doth follow. Numeration is either ſimple or mingled.

Numeration both equalled and proportioned, is ſimple or mingled; and contrariwiſe, both ſimple and mingled Numeration, is equalled or proportioned: Rightly therefore

haue we made ỹ general distinction of Numeration twofold: for so the doctrine of Numeration shall be more truly and plainely perceiued.

2 Simple Numeration is that in which the numbers giuen to be numbred are first numbers of the same kind: and it is of true whole numbers or of true parts. Simple Numeration of true whole numbers is when as the numbers assigned to be numbred are true whole numbers onely.

3 Addition of true whole numbers draweth a line vnderneath the numbers to be added: and then it setteth downe the summe of the added vnder the line drawne. As in this example, where 753 are to be added to 143.

753
143

Here by the 5 verse of ỹ 3 Chap. beginning at the right hand, I adde 3 to 3, whose summe is 6: Wherefore in the first degree I subscribe 6. Then I adde 4 and 5. and they make 9, which I subscribe in the second degree. Againe 1 and 7 are 8, wherefore in the third degree I subscribe 8. The whole example is thus:

753
143
896

In

In this example the ſumme of the ſeuerall degrées was alwaies contained in one figure: If it be contained in two or more, then I kéepe in mind ỳ figures of the higher degrées to be numbred with the degrées following, by the 4 verſe, 3 Chap. Let 790, 889, 952, and 9,843, be giuen to be added: firſt I place the numbers to be added thus:

790
889
952
9,843

Then the ſumme of 3, 2, and 9, is 14: wherefore I ſet 4 vnderneath the line in the firſt degrée, and the 1 I kéepe in mind. Againe the ſumme of 1, 4, 5, 8, and 9 is 27: wherefore I write downe 7, and the 2 I kéepe in mind; in like manner the ſumme of 2, 8, 9, 8, and 7, is 34: wherefore I ſet downe 4, and the 3 I kéepe in mind. Laſtly the ſumme of 3, and 9, is 12: wherefore I write downe 12. The example is thus:

790
889
952
9,843

12,474.

The yeares from the Creation of the world,

vnto the Floud, were 1,656: from the Floud to Moſes 797: from Moſes to Chriſt 1.514: from Chriſt to this day 16.6. The number of yeares from the Creation of the world to this day is ſought. I anſwer, the ſumme of the particulars giuen is 5.583: And therefore from the Creation vnto this day there are paſſed 5.583 yeares. Alſo let theſe particulars following be added. The Addition ſhall be thus:

9,279
389
479
599
689
779
899
989
679
289
189
98
99

15,466.

Here the figures of the firſt degrée added do make 116: Likewiſe thoſe of the ſecond do make 105. In both which, as you ſée, one figure is to be kept in mind, and to be numbred with

with those of the third degrée following. Wherefore in such like sums, where the particulars are many, it shall not be amisse to part them into seuerall companies; then adding each seuerall companie by it selfe, the summes of the seuerals added, shall be the summe of all the particulars assigned.

Againe admit that it were demanded, of what number of Horse & Fote, Darius his army leuied against Alexander of Macedony, did consist. That, out of Quintus Curtius, we thus gather.

Darius saith he, had in his army leuied against Alexander the Great of

1 Persians,	Fote-men,	270,000
	Horse-men,	30,000
2 Medians,	Horse-men,	10,000
	Fote-men,	50,000
3 Bactrians,	Horse-men,	2,000
	Fote-men,	10,000
4 Armenians,	Horse-men,	7,000
	Fote-men,	40 000
5 Hyrcanians	Horse-men,	6.000
Derwickes		40,000
6 The inhabitants about the Caspian sea.	Fote-men,	8,000
	Horse-men,	200
7 Other nations of Asia,	Fote-men,	2,000
	Horse-men,	4.000
8 Grecian stipendaries,		30,000

The summe of Horse and Fote, 509,200.

The army therefore of Darius contained 509,200, besides Bactrians, Sogdians, Indians, with other inhabitants about the Red sea, and some other vnknowne countries, which for haste he could not muster.

4 Subduction of true whole numbers, setteth the number to be subducted vnderneath that from which the Subduction is to be made, then blotting out the numbred, setteth the remaine ouer them.

Let 345 be taken out of 468: first I set the numbers thus:

468
345

Then by the 8 verse, 3 Chap. beginning at the left hand I take 3 out of 4, and there remaineth 1, wherefore 3 and 4 being blotted out I write 1 ouer them. Then I take 4 out of 6, and there remaine 2, wherefore 4 and 6 being blotted out, I write 2 ouer their heads. Againe I take 5 out of 8, and there remaine 3, wherfore 5 and 8 being blotted out, I write 3 ouer them. The example is thus:

1 2 3
468
345

Here the figure to be subducted was neuer greater then the ouermost from which the subtraction

ſubtraction was to be made: if it be greater, then the ouermoſt figure ſhall be increaſed with an vnitie of the former figure. Therefore here we kéepe in mind an vnitie of the nert former degrée numbred, by the 4 verſe, 3 Chap. Out of 6,700 039 there are to be taken 5,900,084, the remaine is demanded. Firſt the numbers to be numbred I write thus:

6,700,039.
5,900,084.

Then I take 5 out of 6, and there remaineth 1: but 9 in the nert degrée cannot be taken out of 7, therefore 5 and 6 being cancelled, I kéepe the 1 in mind. Now I take 9 out of 17, there remaineth 8: but then 8 cannot be taken out of 3, therefore I write ouer their heads 2, and I kéepe one in mind. In like manner I take 0,0,0, out of 10,10,10, & there remaine 10,10,10: but for the ſame cauſe the circles being daſhed I write ouer them 9,9,9, and 1 I kéepe in mind: then I take 8, out of 13, and there remaine 5, which remaine (hauing daſhed 8, and 3) I ſet ouer their heads. Laſtly out of 9, I take 4, & the remaine 5 I ſet ouer their heads, blotting out 9, and 4. The example is thus:

799,955
6,000,039.
5,900,084.

If that the numbers to be subducted, or that from which ỹ Subduction is to be made, be many, first they are to be brought by Addition into one summe: as if you would subduct 465 and 234 out of 905. First thou shalt adde 465 and 234: the summe of the added is 699 which being taken out of 905, the remaine shall be 206.

Suppose the number of Darius his souldiers out of Curtius, to be 509,200, as we found by Addition: He being ouerthrowne, as Iustine in his 11 booke reporteth, lost 61,000, footmen with 10,000 horsemen, beside 40,000 which were take prisoners: how many therefore escaped? I adde ỹ particulars, the summe is 111,000: which subtracted out of 509,200, there remaine 398,200. The example is thus:

398,200
509200
111000

CHAP. VIII.

Of Multiplication and Diuision of true whole numbers.

1 MVltiplication of true whole numbers draweth a line vnderneath the numbers to be multiplied, and then it writeth the products

products of the assigned numbers vnder the right line drawne. Here each figure of the multiplicant doth multiply euery figure of the multiplicand.

Let 873 be multiplied by 58. First I write the numbers giuen thus:

873
58

Then by the 2 verse, 4 Chap. I begin at the right hand, and I multiply the seuerall figures by the 7 verse, 4 Chapter. Therefore 8 times 3 are 24: which 4 I place in the first degrée vnderneath the line, the 2 I kéepe in mind by the 4 verse, 3 Chapter: then 8 times 7 are 56: vnto which I adde the 2 in mind, and the summe is 58: which 8 I write downe in the second degrée: the 5 I kéepe: likewise 8 times 8, are 64: which with the 5 in minde are 69. Therefore I set downe the whole 69. Againe, I multiply 5 by 3, the product is 15: which 5 I set downe in the second degrée right vnderneath 5, the multiplicant: the 1 I kéepe: then 5 times 7 are 35, to which I adde 1, and the summe is 36: which 6, I set downe, kéeping the 3 in memory. Lastly, 5 times 8 are 40, to which I adde the 3 kept, the summe is 43: which whole summe I likewise set downe. I adde the product of the two seuerall figures of

the

the multiplier by the multiplicand, and the ſumme 50,634, I affirme to be the product of 873, by 58, the two numbers aſſigned. The example is thus: 873

58

6984

4365

50,634

If the firſt figures either of one or both of the numbers giuen to be multiplied be Cyphers; then thoſe Cyphers neglected, and the other multiplied, the compound of their products ioyned to thoſe neglected cyphers ſhal be the deſired product. 78,000 are to be multiplied by 650. The cyphers neglected, I ſet the figures to be numbred thus:

78

65

Then multiplying 78, by 5, and by 6, I make 390, and 4 680, the ſumme of which is 5.070: to this I put the foure cyphers neglected, the example is thus.

78

65

390

468

50,700,000

Here-

Hereupon it followeth that if the laſt figure of the multiplicand be an vnity, the other cyphers, then thoſe cyphers ſet before the multiplicand, do yéeld the product of the numbers aſſigned: If 750, were to be multiplied by 1,000, I ſay their product is 750,000: the reaſon is manifeſt, for that an vnity in Multiplication doth not alter. Let the vſe of Multiplication be ſhewed in a thing numbred: The number of dayes ſince the Creation vnto this preſent day being the 28 of March, 1594. is ſought. The number of yeares we found at the 3 verſe, 7 Chapter, to be 5561, the numbers of dayes in a yeare are granted 365: therefore I multiply 5,561, by 5. 6. and 3. and I make 27805: 333660, and 1668300, the ſumme of which is 2,029,765: I adde 1390, the number of leape yeares paſſed ſince the beginning of the world, as alſo 360, the number of dayes paſt ſince the 25 of March, on which day ſome thinke the world to haue bene made. The ſumme of all which 2031,515, I affirme to be the number of daies ſince the Creation of the world: The example is thus.

5,561

5,561
365
27805
33366
16683
2,029,765
1390
360
2,031,515

2 Diuision of true whole numbers placeth the Diuidend betweene the Diuisor and the quotient: It setteth the Diuisor toward the left hand; and findeth the quotient by Subduction of the diuisor from the Diuidend as oft as may be. Here euery figure of the Diuisor is equally subducted from the Diuidend:

Betwéne the] diuerse Authors haue vsed diuerse formes and manners of deuiding: some for euery particular Diuision do place anew the Diuisor vnderneath the Diuidend, and vnderneath it subscribeth the product of the Diuisor by the quotient, so that the Diuision being done, the Diuisor may not easily be distinguished from the product. Others do neither write downe the Diuisor nor the product made; this is worse for ỹ memorie then the former. Orontius and others haue vsed other

ther waies; but none of them may be compared vnto this of Salignacus, for that euery particular Diuision being so plainely distinguished, may afterward be easily examined, if any doubt be made: whereas those other cannot; the one for the confusion of ỹ figures, the other for the want of them. Let 724 be deuided by 362, here I take 3 out of 7 twise, and there remaineth 1: in like manner I subtract 6 out of 12 twise, and there remaineth nought: Lastly, I take 2 out of 4 twise, and there remaineth nought: Wherefore the quotient of 724 by 362 shall be 2: the example is thus:

1
362) 724 (2

Diuision of whole numbers doth greatly trouble the memory, more then the rest of the Numerations: And therefore they which do hardly trust their memory (and surely here few do trust it) do straight waies examine the diuision conceiued in mind by the Multiplication of the quotient by the Diuisor, and the Subduction of the product from the Dividend, as thou maist sée in this example.

362) 724 (2
724

If the first figure of the Diuisor be greater then

then the first figure of the Diuidend, we take it out of the two former figures of the Diuidend: 768 are to be deuided by 96: here 9 is greater then 7, therefore I take 9 not out of 7, but out of 76: the example is thus.

96) 768 (8
768

If the quotient be to be written by many figures, then the former Diuisions being done, we do cancell the next following figure of the Diuidend so, that we may write it againe in a higher place in the same degree. Examples in which the Diuisor is a part of the Diuidend: 1,008 are to be deuided by 28, the Diuision is thus.

1̸68
28) 1̸008 (36
84
1̸68

Which example (as being somewhat more harder) I do thus more particularly declare: The first figure of the Diuisor being greater then the first of the Diuidend, I take it out of the two first figures: that is, I take 2 of 10 fiue times, and there remaine nought: but I must take 8 also so many times out of ỹ same Diuidend: which I cannot do out of 0: therefore I subduct 2 out of 10 but 4 times, & there remaine

remaine 2: but yet I cannot ſubduct 8 ſo many times out of 20, therefore I ſubduct 3 out of 10 but 3. Now I multiply 3 by 28, and I make 84 which I place ſo as the firſt figure of the product do anſwer vnto the firſt of the particular Diuidend: then ſubtracting 84 out of 100, there remaine 16, which I write ouer their heads: and therewithall I cancell 8 the firſt figure of the Diuidend, and place him againe ouer the ſame degrée. Now I ſubduct 2 out of 16, which I may do 8 times, but then I muſt take 8, the other figure of the Diuiſor as many times out of the figures remaining, which I cannot do; therefore I find it but 7: now 7 by 2, make 14, and 14 out of 16, there remaine 2. Neither can I yet take 8 ſeuen times out of 28: therefore I take 2 out of 16 but 6 times; then I multiply 28 by 6, and the product is 168: which I ſubtract from 168. and there remaine 0, which ſignifieth that the Diuiſor is a part of the Diuidend, by the 5 verſe, 5 Chapter. Wherefore by the ſame verſe, I affirme 28, to be $\frac{1}{36}$ of 1,008.

Let 42,082 be deuided by 53. The Diuiſion is thus:

```
      2 1 2
      4 9 8
53) 4 2 0 8 2 (794
    3 7 1
      4 7 7
        1 1 2
```

Examples where the Diuisor is parts of the Diuidend. 873 l are to be deuided amongst 382 sharers. The Diuision is thus:

```
      1 0 9
382) 8 7 3 (2 109/382
    7 6 4
```

Therefore here, by the 6 verse, 5 Chapter, 382 shall be $\frac{382}{873}$ parts of 873. And the Diuision signifieth ỹ each mans part shall be ỹ l, and moreouer also $\frac{109}{382}$ l, that is v s̄ vy d̄ with some very small portion more.

Item, a booty of 43.165 l, is to be deuided amongst 76 souldiers: The question is, what euery mans part shall be? The Diuision is thus:

```
        7 3
      6 0 5
    5 1 6
76) 4 3,1 6 5 · (5 67 71/76 l
    3 8 0
      4 5 6
        5 3 2
```

This

This Diuision doth signifie that each mans part shal be 567 l. with $\frac{73}{76}$ parts of one pound, which by Reduction, as afterward shall be taught, doth containe xix s̄ y d̄ ob. with some small portion more.

If the first figure of the Diuisor be an vnitie, and the other cyphers, then so many figures being taken from the latter end of the Diuidend, as there be cyphers in the Diuisor, the Diuision shall be done.

If the Diuisor be a part of the Diuidend, the Diuision is thus:

100) 685,000 (6850

If the Diuisor be parts of the Diuidend, the Diuision shall be thus:

1000) 5,000,693 (5,000 $\frac{693}{1,000}$

This Diuision signifieth that each seuerall mans part shall be 5,000 l. xiij s. x d. q. and somewhat more.

CHAP. IX.

The properties of true whole numbers may be drawne out of their Numeration.

THe properties of true whole numbers are considered partly in numbers by themselues

themselues, partly in numbers betweene themselues. In numbers considered by themselues we haue two distinctions. The first is, A number is euen or odde: An euen, or equall number is that which two doth measure.

So 2, 4, 6. are euen numbers.

2 An euen or equall number is either equally equall, or vnequally equall. Equally equall is an euen number, each of whose measures of many is an euen number.

So 2, 4, 8, 16, 32, are equally equall numbers. Such are all numbers which are made by doubling of two.

Elian in his booke which he wrote of the ordering of battels, saith that this kind of number was first thought on and obserued for his great vse in warlike affaires, as being most commodious for the changing and altering of the formes of battels.

3 An vnequally equall number, is that whose measure of many is sometimes euen sometimes odde.

Such are 6, 12, 14.

4 An odde number is that which two cannot measure.

Such are 1, 3, 5, 7, 9, 15.

To this place refer the telling of a number thought; wch doth especially consist of this third

distinction

diſtinction of a number into euen and odde.

5 Againe, a number conſidered by it ſelfe is either ſimple or compound. A ſimple number is that whoſe greateſt meaſure of many is no other then it ſelfe.

So 1, 2, 3, 5, 7, 11, are ſimple numbers.

6 A compound number is that whoſe greateſt meaſure is another number of many beſide it ſelfe.

Such are 4, 6, 8, 9.

7 Numbers conſidered betweene themſelues, are alſo ſimple or compound betweene themſelues. Simple numbers betweene themſelues, are thoſe whoſe greateſt common meaſure is an vnitie.

Such are 5 & 4, 10 and 21, for that no number of many can meaſure 5 and 4: nor 10 & 21.

8 Compound numbers betweene themſelues are thoſe, whoſe greateſt common meaſure, is a number of many.

Such are 3 and 3: 5 and 15: 4 and 6.

CHAP. X.

Of the properties of numbers betweene themſelues.

1 IF two numbers be firſt betweene themſelues, the product of them ſhall be the

least Diuidend of them.

Let 3 and 5 be simple numbers betweene themselues, and the product of them be 15, I say that 15 is the least Diuidend of 3 and 5, the numbers assigned.

2 If a number shall measure the assigned by simple numbers betweene themselues, it shall be the greatest common measure of the assigned.

Let 6 measure 24. 54. and 72. by 4, 9, and 12, simple numbers betweene themselues: I say that 6 is the greatest common measure of 24, 54, and 72.

3 If a simple number of many shall measure the product of two numbers, it shall at leastwise measure one of the assigned.

Let 4 and 6 make 24: and let 3 measure 24, by 8: I say that 3, the simple number of many shall measure one of the assigned 4, or 6.

It may sometimes measure them both.

CHAP. XI.

Of the finding of all the measures of an assigned compound number.

1 THe finding out of all the measures of an assigned compound number hath two parts: the first part is thus;

2 If

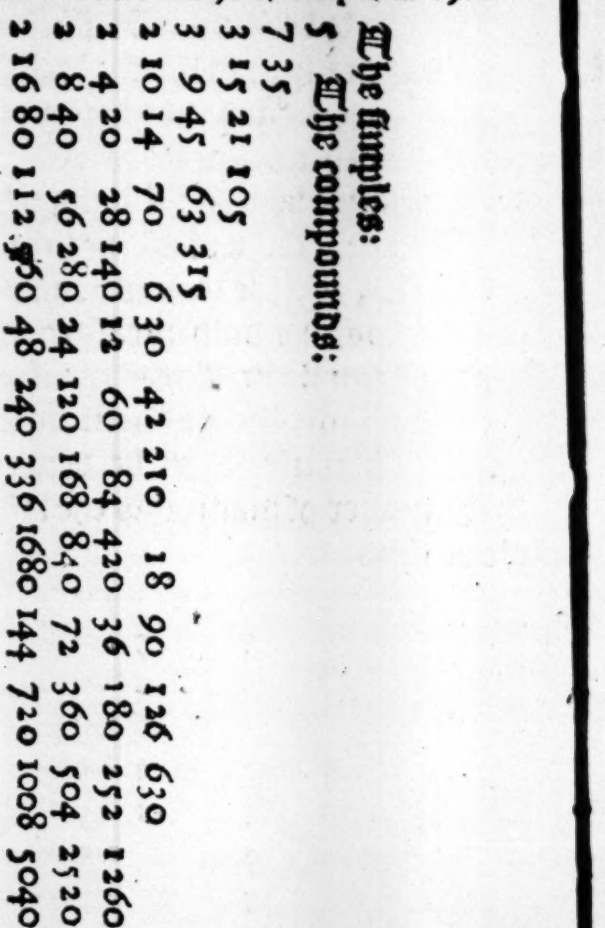
2 If first numbers of many from the first shall continually measure both the assigned and the quotient of the assigned, and afterward the quotient of the quotient as oft as may be, such like measures, the last quotient & an vnity, shall be all the simple measurers of the assigned.

All the simple measurers of the compound 5,040, are to be found. Of all the simple measurers of many, 2 is the the least, by which I deuide 5,040, and the quotient is 2,520: the which 2 doth measure by 1,260: the which 2 doth measure by 630: the which 2 doth measure by 315. Now of all the simple measures measuring 315, the least is 3, and the quotient is 105: which 3 doth measure by 35. Lastly of all the simple measures measuring 35, the least is 5. and the quotient by it is 7: which quotient because it can be deuided of none other number of many, it is the last. The example is thus:

	5,040
2	2,520
2	1,260
2	630
2	315
3	105
3	35
5	7

3 The second part is thus: If the simple numbers of many found from the last saue one, shall multiply once the following simple numbers, and all the products of them, the products shal be all the compounds measuring the assigned.

All the compounds measuring the assigned 5,040, are to be found. The example is thus:



If therefore a Captaine hauing 5,040 souldiers, be desirous to know how many waies he may rightly embattell them, I answer by this rule, 30 sundry manner of waies: that is, halfe the number of the measures shall be the desired number. The orders or rankes of souldiers are thus:

1	5	7	25	3	15	21	
5040	1008	720	144	1680	336	240	
105	9	45	60	315	2	10	14
46	560	112	84	16	2520	504	360
70	6	30	42	210	18	90	126
72	840	168	120	24	280	56	40
560	4	20	28	140	12	60	
9	1260	252	180	36	420	84	

CHAP. XI.

Of the generall finding both of the greatest common Measure and least common Diuidend.

1 OF the common Measures commonly the greatest is sought, but of the common Diuidend the least.

2 The finding out of the greatest common Measure is thus: If of two numbers giuen you

shall take the one of them out of the other as oft as may be, and shall continue the like worke in the subtracted, and in the remaine as oft as you may, the last subducted shall be the greatest common measure of them both.

We prescribe this to be done by Subduction, which may also be done by Diuision: for Diuision is a manifold Subduction.

The greatest common measure of 40 and 16, is to be found: First I subduct 16 out of 40 twise, and the remaine is 8: which I subduct out of 16, twise, & the remaine is nought: therefore I say that 8 the last subducted shall be the greatest common measure of 40, and 16. The example is thus:

By Subduction:

40, 16
16, 8
8

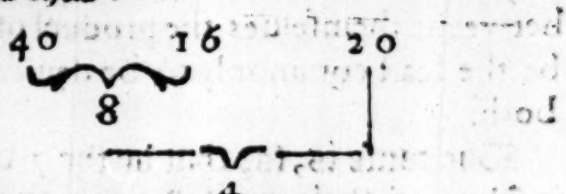
By Diuision:

8) 40, (2
32
8) 16 (2
16

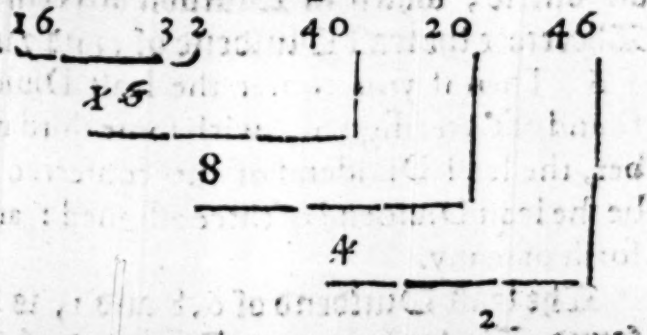
Then if thou doest conferre the greatest common measure of two assigned with some third number, the greatest common measure of the conferred shall be the greatest common measure

measure of the three assigned: and so forth in more assigned.

The greatest common measure of 40, 16, and 20, is desired: The greatest common measure of 40. and 16, was found to be 8. Therefore in like manner I conferre by the 2 verse, 8 and 20; and I find 4 to be their greatest common measure: therefore I affirme by this verse, 4 to be the greatest common measure of the three assigned, 40, 16, and 20. The example is thus:



Another of more assigned:



4 The finding out of the least common Dividend is on this sort: If of two numbers assigned thou shalt deuide the one of them by their greatest common measure, and shalt multiply the

the other by the quotient, the product shall be the least common Diuident of the assigned.

I seeke the least common Diuident of 6 and 8. The greatest common measure of 6 and and 8 is 2, by the 2 verse. I deuide 6 by 2, and the quotient is 3, by which I multiply 8, and the product is 24: which is the least Diuident of 6 and 8. Or else I deuide 8 by 2, and the quotient is 4, by which I multiply 6, and the product is 24. Therefore

5 If the two numbers assigned be simple betweene themselues, the product of them shal be the least commonly to be deuided of them both.

The cause is, for that by the 7 verse of the 9 Chapter, their greatest common measure is an vnitie, which in Diuision altereth not. Therefore the least Diuident of 5 and 7 is 35.

6 Then if you confer the least Diuident found of two assigned, with some third number, the least Diuident of the conferred shall be the least Diuident of three assigned: and so forth of many.

The least Diuident of 6, 8 and 15 is to be found. The least common Diuident of 6 and 8 is 24, by the 4 verse: which 24, the third number 15 cannot measure; therefore by the same 4 verse, I find 120 to be the least common Diuident

uident of 24 and 15. And therefore by this verse I affirme 120 to be the least common Diuident of 6,8 and 15. The example is thus:

6 8 15

24

120

CHAP. XII.

Of the Numeration of true parts generally, and of the Reductions adioyned to it.

1 THe Numeration of true parts is when the assigned to be numbred are true parts onely.

2 To this Numeration is adioyned a Reduction of bounds and of parts. The Reduction of bounds deuideth the bounds compounded betweene themselues, by their greatest common measure, and taketh the quotients for the assigned bounds.

So we deuide $\frac{4}{12}$ being compound bounds, by their greatest common measure 4, and we find $\frac{1}{3}$ proportionall vnto $\frac{4}{12}$, the parts assigned. So for $\frac{6}{14}$ we may take $\frac{3}{7}$. And so for 9 and 21 we may take 3 and 7.

3 Re-

3 Reduction of parts is either vnto whole numbers or vnto parts. Reduction vnto whole numbers is twofold. The first doth take the quotient of the Numerator deuided by the Denominator for the solitary parts.

Such a like quotient is taken for solitarie parts, because it is the whole consisting of them. So the whole of $\frac{2}{2}$ is 1, the whole of $\frac{6}{3}$ is 2. Therefore for $\frac{2}{2}$ and for $\frac{6}{3}$ we take 1 and 2.

4 The second doth take the Numerators of the parts assigned, being of the same denomination betweene themselues, for the parts themselues.

For such like Numerators are proportionall vnto the parts assigned. Therefore for $\frac{4}{7}$ and $\frac{5}{7}$ we take 4 and 5.

5 Reduction vnto parts, doth take equall parts of the same denomination, to the parts assigned of diuerse denominations, for the parts assigned.

This Reduction may be done many waies, but most briefly thus: we take the least Diuidend of the assigned Denominators for the common Denominator of the parts sought: but for their Numerators we take the fourth proportionall to the Denominator of ỹ parts assigned, the least Diuidend found, and the Numerator of the parts.

So

So for $\frac{5}{6}$ $\frac{4}{9}$ $\frac{7}{36}$ $\frac{1}{2}$ and $\frac{1}{4}$: we take $\frac{30}{36}$ $\frac{16}{36}$ $\frac{7}{36}$ $\frac{18}{36}$ $\frac{9}{36}$. The least Diuidend is to be found by the 4 verse of the 11 Chapter. The fourth proportionall is found by the 10 verse of the 6 Chapter. Now $\frac{30}{36}$ are equall to $\frac{5}{6}$ by the 7 verse of the 6 Chapter: for if the bounds of parts be proportionall, the parts themselues are equall.

CHAP. XIII.

Of the equalled Numeration of true parts.

1 THe equalled numeration of true parts doth number Numerators onely of the parts of the same denomination: and then putteth the common Denominator of the parts assigned, vnderneath the number found.

2 Addition of true parts doth adde their Numerators, as followeth:

Unto $\frac{2}{7}$ adde $\frac{3}{7}$ and $\frac{4}{7}$; the whole shall be $\frac{9}{7}$, that is by the 3 verse of the 13 Chapter $1\frac{2}{7}$.

Unto $\frac{2}{4}$ adde $\frac{3}{10}$: here if you reduce ẏ bounds before addition, the addition shall be thus:

$$\begin{array}{ccc} 1 & 3 & 5 \\ \cancel{2} & \cancel{3} & \cancel{1} \\ \hline 4 & \cancel{1}\cancel{0} & \cancel{2} \\ 2 & \cancel{1} & \cancel{5} \\ & \underbrace{\qquad\quad}_{10} & \end{array}$$

The whole is $\frac{8}{10}$ that is, $\frac{4}{5}$.

If

If you defer the Reduction vnto the latter end of Addition, the Addition shall be thus:

$$\begin{array}{cc} 10 & 6 \\ \cancel{2} & \cancel{3} \\ \hline \cancel{4} & \cancel{10} \\ \cancel{5} & \cancel{2} \\ \hline \multicolumn{2}{c}{20} \end{array}$$

The whole is $\frac{16}{20}$ that is $\frac{4}{5}$.

But this second way is shorter.

Vnto $\frac{5}{8}$ adde $\frac{4}{9}$ $\frac{7}{12}$ $\frac{5}{6}$. Here by the 5 verse of the 13 Chapter, for the assigned parts we take $\frac{45}{72}$ $\frac{32}{72}$ $\frac{42}{72}$ and $\frac{60}{72}$: their summe is $\frac{179}{72}$, that is, by the 3 verse of the 13 Chapter, $2\frac{35}{72}$.

3 Subduction of true parts, taketh the one of their Numerators out of the other, as followeth.

Take $\frac{3}{9}$ out of $\frac{5}{9}$, the remaine shall be $\frac{2}{9}$: out of $\frac{2}{4}$ take $\frac{3}{10}$, the remaine shall be $\frac{4}{20}$: or by Reduction of bounds (by the 2 verse of the 13 Chapter) $\frac{1}{5}$. The example is thus:

$$\begin{array}{cc} 10 & 6 \\ \cancel{2} & \cancel{3} \\ \hline \cancel{4} & \cancel{10} \\ \cancel{5} & \cancel{2} \\ \hline \multicolumn{2}{c}{20} \end{array}$$

Here therefore as in Addition, Reduction of

of the bounds is to be deferred vnto the end of Numeration.

Take $\frac{2}{12}$ out of $\frac{5}{33}$ and $\frac{7}{33}$. First by the 5 verse of the 13 Chapter, for the assigned parts take $\frac{42}{132}$ $\frac{22}{132}$ and $\frac{20}{132}$. The summe of $\frac{22}{132}$ and $\frac{20}{132}$ is $\frac{42}{132}$ by the 2 verse of this Chapter. Then $\frac{42}{132}$ taken out of $\frac{42}{132}$, nothing shall remaine. Therefore $\frac{2}{12}$ taken out of $\frac{5}{33}$ and $\frac{7}{33}$ shall leaue nothing.

CHAP. XIIII.

Of the proportioned Numeration of true parts.

1 IN the proportioned Numeration of true parts, the number found is alwaies vnequal vnto one of the assigned.

In multiplication of true parts the multiplier is lesser then an vnity: and therefore by the 2 verse of the 4 Chapter, here the product shall be lesser then the multiplied. In Diuision an vnity is greater then the Diuisor, and therefore here by the 1 verse of the 5 Chapter, the quotient shall be greater then ỹ deuided.

2 Multiplication of true parts multiplieth the bounds of the same kinde betweene themselues.

By

By bounds of the same kind, we vnderstand the Numerator and Numerator; againe, the Denominator and Denominator. By bounds of diuerse kinds, their Numerator and Denominator. Therefore the product of $\frac{3}{4}$ by $\frac{5}{7}$, shall be $\frac{15}{28}$.

3 If the bounds of diuerse kinds as well of the same parts as of diuers, be simple betweene themselues, the products of the bounds of the same kind between themselues shall be simple.

Therefore here before Multiplication we reduce the bounds of diuerse kinds compounded, by the 2 verse of the 13 Chapter. Therefore in the Multiplication of $\frac{2}{3}$ by $\frac{5}{6}$, the bounds being reduced to $\frac{1}{3}$ and $\frac{5}{5}$, thou shalt make $\frac{5}{9}$.

Therefore

4 If the bounds of diuerse kinds of the parts to be multiplied be partly equall, partly vnequall, the equall neglected, the Multiplication shall be done.

For the equall are reduced vnto vnities: & an vnitie in Multiplication doth not alter. Wherefore the product of $\frac{4}{5}$ by $\frac{5}{7}$ shall be $\frac{4}{7}$. And the product of $\frac{2}{3}$ by $\frac{3}{5}\frac{5}{7}\frac{7}{8}$ shall be $\frac{2}{9}$.

5 Diuision of true parts for the assigned parts, deuideth whole numbers proportionall vnto them.

Deuide $\frac{7}{10}$ by $\frac{3}{10}$ the quotient of this Diuision

tion shall be the quotient of 7 by 3. that is $2\frac{1}{3}$. For $\frac{7}{10}$ and $\frac{1}{10}$ are proportionall vnto 7 and 3, by the 4 verse of the 13 Chapter.

If the parts to be deuided be of diuerse Denominations, they are to be reduced by the 5 verse of the 13 Chapter. As if $\frac{7}{8}$ were to be deuided by $\frac{1}{5}$: first I reduce them to the same denomination, then the example is thus:

35 24

$\frac{7}{8}$ $\frac{3}{5}$

5 8

40

11

24) 35 (1 $\frac{11}{24}$

24

Or else (the parts being of diuerse denominations) the simple Numerators shall be found by this rule following:

6 If the bounds both of the same kinds and of diuerse kinds of the same parts, be simple betweene themselues, the products of the Numerators by the each other Denominator of the parts assigned, shall be simple betweene themselues.

$\frac{7}{8}$ are to be deuided by $\frac{3}{5}$: the Numerators shall be found the same as at the 5 verse of this Chapter. The example is thus:

7 3

8 5

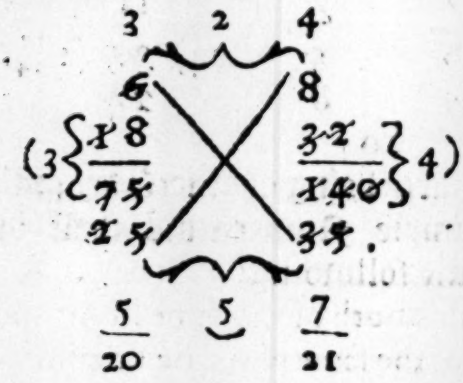
24 35

11

24) 35 1 $\frac{11}{24}$

24

If that the bounds be compound betweene themselues, they are to be reduced to the simple betweene themselues by the 2 verse of the 13 Chapter. Therefore in the diuision of $\frac{18}{77}$ by $\frac{32}{140}$, the bounds being reduced, we deuide the numbers found 21 and 20 for the assigned parts, and we take the quotient $1\frac{1}{10}$ for the quotient of the assigned parts. The example is thus:



Therefore

7 If the Numerator of the parts to be deuided be equall vnto the Numerator of the deuiding parts, the Numerators neglected, the Denominator of ỹ deuiding parts shall be deuided by the Denominator of the parts to be deuided.

For equall numbers are reduced vnto vnities: and an vnity in multiplying doth not increase. If therefore $\frac{4}{7}$ were to be deuided by $\frac{4}{5}$, the quotient shall be $\frac{5}{7}$.

CHAP.

CHAP. XV.

Of mingled Numeration in generall, and of mingled equalled Numeration.

1 MIngled Numeration followeth, in which the assigned to be numbred are numbers of diuerse kinds.

2 To the mingled Numeration beside the former Reductions there is also adioyned the Reduction of whole numbers, which for true whole numbers taketh secondary parts equall vnto them.

Here the Denominator of the parts is giuen, for the Numerator of which, we take the product of the whole numbers by the assigned Denominator. Therefore when we reduce 1 vnto seconds, 5 vnto twelfths, we take $\frac{1}{1}$ and $\frac{5}{1}$ for 1 and 5.

3 Mingled equalled Numeration doth number seuerally the whole numbers by themselues, and the parts by themselues.

4 Mingled Addition addeth the assigned, as followeth.

Vnto 5, adde $4\frac{7}{12}$, the summe shall be $9\frac{7}{12}$: vnto $4\frac{7}{12}$ adde $7\frac{5}{12}$. First the summe of the parts is $\frac{12}{12}$, that is $1\frac{17}{12}$. Now to 1 adde 4 and 7, the summe of the added shall be 12: wherefore the summe of the whole Addition is $12\frac{17}{12}$.

Unto 48 ℓ. 17 s̃ 8 d̃, adde 54 ℓ 6 s̃ 9 d̃, and 18 ℓ 13 s̃ 10 d̃. The Addition shall be thus:

48 ℓ	17 s̃	8 d̃
54	6	9
18	13	10
121	18	3

5 Mingled Subduction taketh the one out of the other, as followeth.

Take $2\frac{7}{8}$ and $1\frac{45}{56}$, out of $6\frac{5}{7}$. Here first thou shalt adde $2\frac{7}{8}$ & $1\frac{45}{56}$, & the summe shall be $4\frac{19}{28}$. Then take 4 out of 6, the remaine shall be 2. Againe $\frac{19}{28}$ out of $\frac{5}{7}$ and the remaine is $\frac{1}{28}$: so the whole remaine of the Subduction propounded is $2\frac{1}{28}$. If the parts to be subducted be greater then those from which ẏ Subduction is to be made, then we borrow an vnity of the other whole numbers, which we reduce to parts of the same denomination with the assigned. Take $2\frac{3}{4}$ out of $9\frac{1}{2}$. Here first thou shalt take 2 out of 9, and there shall remaine 7: but because that $\frac{3}{4}$ cannot be taken out of $\frac{2}{4}$, therefore for 7 I set downe 6 onely, and then out of $\frac{6}{4}$, I take $\frac{3}{4}$ and there remaine $\frac{3}{4}$. And so the whole remaine is $6\frac{3}{4}$. Out of 19 ℓ 9 s̃ 5 d̃, take 18 ℓ 15 s̃ 8 d̃, ẏ Subduction shall be thus:

ℓ	13	9
~~19~~	~~9~~ s̃	~~5~~ d̃
~~18~~	~~15~~	8

CHAP.

CHAP. XVI.

Of mingled proportioned Numeration.

1 MIngled proportioned Numeration numbreth the parts with the whole numbers ioyntly together.

2 This Numeration doth follow the order of the proportioned Numeration of true parts: and therfore it setteth an vnity vnderneath true whole numbers.

3 Mingled multiplication multiplieth the bounds of the same kind, as followeth.

Would you know what the value of $\frac{5}{8}$ of 60 ℔ is? multiply $\frac{5}{8}$ by $\frac{60}{1}$, thou shalt make $\frac{75}{2}$, that is by the 3 verse of the 13 Chapter, $37\frac{1}{2}$: therefore $\frac{5}{8}$ of 60 ℔ shall be $37\frac{1}{2}$ ℔.

So $\frac{7}{9}$ of a circle deuided into 360 degrées, is $\frac{2520}{9}$, that is, 280 degrées.

Reduction of parcels vnto parts, is a mingled multiplication, whereby we multiply parts by secondary whole numbers. A man making his will bequeathed to his first sonne $\frac{1}{3}$ of his goods, to the second $\frac{1}{3}$ of ỹ moueables remaining, to the third the residue of his substance. It is demanded of the second sonnes portion, what part it is of the whole substance? Multiply $\frac{1}{3}$ by $\frac{4}{5}$ and thou shalt make $\frac{4}{15}$, which do signifie $\frac{1}{3}$ $\frac{4}{5}$ to be $\frac{4}{15}$ of the whole goods. So

 $\frac{2}{3}$

$\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{7}$ are $\frac{2}{7}$ by the 4 verse of the 15 Chapter.

Multiply $5\frac{2}{3}$ by $3\frac{3}{7}$. First by the 2 verse of the 16 Chapter, for the bounds giuen thou shalt take $\frac{17}{3}$ & $\frac{17}{7}$: which multiplied betweene themselues, do make $\frac{289}{15}$, that is, $19\frac{4}{15}$.

4 Mingled Diuision findeth the quotient, as followeth.

$\frac{2}{7}$ are to be deuided by 2: here thou shalt deuide $\frac{2}{7}$ by $\frac{2}{1}$; the quotient by the 7 verse of the 15 Chapter shall be $\frac{1}{7}$.

Deuide $36\frac{60}{169}$ by $7\frac{5}{13}$. Here first thou shalt reduce the assigned to whole numbers 64 and 13. Then the quotient of 64 by 13 is $4\frac{12}{13}$.

The end of the first Booke.

THE

THE SECOND BOOKE OF ARITHMETICKE.

CHAP. I.

1 THE first part of Arithmeticke hath declared the Notation & Numeration of numbers: the second shall declare their comparison in quantity & quality.

2 Comparison in quantity is, when we compare two bounds onely betweene themselues: the first is called the former, the second the follower.

3 Comparison in quantity is of equality or vnequalitie. Comparison of equality is that whose bounds are equall betweene themselues. **So the comparison of 1 to 1, of 2 to 2, are of equality.**

4 Comparison of vnequality is that whose bounds are vnequall betweene themselues. If the forebounds shall be greater or lesser then

the following bound, the comparison shall be of greater or lesser vnequality.

So the comparison of 3 vnto 2, is a comparison of greater vnequality: the comparison of 2 vnto 3, of lesser vnequality.

5 Againe comparison in quantity, is the difference or the reason.

6 The difference is the remaine which is found by Subduction of the lesser from the greater.

So the difference of 3 from 2 is 1, in comparison of greater vnequality: the difference of 3 from 8 is 5, in comparison of greater vnequality. Therefore the Numeration of the differences, is the numeration of ỹ remaines. If the difference of 3 from 2, be to be added to the difference of 3 from 8, thou shalt adde 1 to 5. If this to be multiplied by that, thou shalt multiply 5 by 1. The same is to be said of the Subduction and Diuision of the differences.

7 The Reason is the quotient of the former deuided by the follower.

So, when it is asked what the reason is of 2 into 2, of 2 into 1, and of 2 into 3; we shall answer, the quotients are 1, 2 & $\frac{2}{3}$, that is, the reason of equality, double and vnder once and halfe. Therefore as before the Numeration of the differences was called the Numeration of

the

the remaines, so now the Numeration of reasons we call ỹ Numeration of the quotients. Notwithstanding oftentimes we number the bounds of ỹ reasons themselues for the quotients as if they were parts: that is to say, we take the former bounds of the reasons as Numerators, the following bounds as the Denominators of the parts: for so the Numeration of the bounds doth altogether answer and is agreeable to the Numeration of the quotients.

8 The former bounds of two reasons, and likewise the following bounds are called Agreeable, but the formers of one reason and the followers of another, Disagreeable.

As 1 is to 2, so is 4 to 8; here 1 and 4: againe 2 and 8, are agreeable bounds: contrariwise 1 and 8, Againe 2 and 4, disagreeable.

9 If it be a reason of greater vnequality it is either simple or manifold. Simple is when the following bound is contained once in the former, and ouer and aboue that something lesse then the former.

10 It is ouerparticular, or ouerpartient. Ouerparticular is that in which the follower being taken out of the former, there remaineth a part of the follower.

If there remaineth $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$, the ouerparticular reason

reason is called sesquialtera, sesquitertia, sesquiquarta, i, once and one halfe, once and one third, once & one fourth: such are the reasons of 3 vnto 2, of 4 vnto 3, of 5 vnto 4.

11 Ouerpartient is, in which the follower being takē out of the former, there do remaine parts of the follower.

If the parts of the follower remaining be $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, the ouerpartient reason is called ouer two thirds, ouer three fourths, or ouer foure fifts. Of this kind are the reasons of 5 to 3, of 7 to 4, of 9 to 5.

12 The manifold reason is, when the following bound is contained in the former many times: and it is iust or ouerplus.

13 Iust manifold is, when the former containeth the follower many times absolutely.

Of this kind is the reason of 2 to 1, of 3 to 1, of 4 to 1. The first is called double, the second treble, the third fourefold.

14 Ouerplus manifold is a manifold reason, when the follower being taken out of the former as oft as may be, there doth remaine a part or parts of the follower: there the reason is called Manifold ouerparticular: here Manifold ouerpartient.

So the reasons of 5 to 2, of 10 to 3, of 17 to 4, are called double halfe other, treble one third,

hird, and fourefold one fourth. So the reasons of 8 to 3, of 15 to 4, of 24 to 5, are called double ouer two thirds; treble ouer three fourths; and fourefold ouer foure fifts.

15 If the reason be of lesser vnequality, it doth retaine the name of contrary reason, this word, vnder, being set before it.

So the reasons of 2 to 3, of 3 to 5, of 2 to 5, and of 3 to 8, vnder halfe other, vnder two thirds, vnder double ouer one halfe, vnder double ouer two thirds. So 1 to 2, and 3 to 9, are vnder double and vnder treble.

But in the expressing of reasons, for the speciall names of reasons oftentimes we pronounce their bounds. So the reason of the compasse vnto his midline is said to be lesse then the reason of 22 vnto 7. The reason of the distance betweene the pillars of Hercules and India vnto the distance betweene Æthiopia and the fenne of Meote, greater then the reason of 5 vnto 3. The reason of a circle vnto the square of his midline, is the reason of 11 vnto 14.

CHAP.

CHAP. II.

Of comparison in quality, and especially of proportion.

1 COmparison in quality, is a comparison of comparison in quantity of the same kind.

Of the same kind I say, to signifie that both of the compared comparisons be either the difference or reason.

2 Comparison in quality is either proportion or disproportion. Proportion is that when the quality of the comparisons in quantity is the same.

The same quality is here vnderstood, when either the differences or reasons are the same betweene themselues.

3 Proportion is either disioyned or continuall. Disioyned is that whose second and third bounds are diuerse: And it is foreward or backward.

4 Forward or direct proportion is that, when the comparison of the first vnto the second, is the comparison of the third vnto the fourth.

As in these examples 1, 3, 5, 7, and 1, 4, 8, 32. As 1 differeth from 3, so 5 differeth from 7: and as 1 is vnto 4, so is 8 vnto 32.

5 If two rankes of bounds be equall in number

umber; and in the first ranke the comparison f the first bound vnto the second, of the se- ond vnto the third, of the third, vnto the ourth, and so forth; be in the second ranke he comparison of the first vnto the second, of he second vnto the third, of the third vnto he fourth, and so foorth; the outmost bounds hall be directly proportionall.

Such like proportion is commonly called Ordered equation, because the vpper row of bounds is taken like vnto the lower. As 1 differeth from 2, so 6 differeth from 7: and as 2 from 8, so 7 from 13, and as 8 from 12, so 13 from 17.

Proportion of	1	2	8	12
the differences:	6	7	13	17

Therefore as 1 differeth from 12, so 6 differeth from 17.

Proportion of	2	8	12	24
the reasons:	7	28	42	84

Therefore as 2 is to 24, so is 7 vnto 84.

6 Backward proportion is a disioyned proportion, when the comparison of the first bound vnto the second, is the comparison of the fourth vnto the third.

As in these examples 1, 3, 7, 5, and 1, 2, 16, 8: as 1 differeth from 3, so 5 differeth from 7; and as 1 is to 2, so is 8 to 16. Therefore this proportion

proportion doth differ from forward or ou
right proportion onely by the placing of t
bounds. Others do thus place the bounds
7, 3, 5, and 1, 16, 2, 8, and so they make th
comparison of the first bounds vnto the third
the comparison of the fourth vnto the second
But ours is shorter and easier.

7 If two rankes of bounds be equall i
number, and the comparison in the first rank
of the first vnto the second, of the second vnto
the third, of the third vnto the fourth, and so
forth; be in the second row the comparison o
the second to the first, of the third to the se
cond, of the fourth to the third, and so forth
the outmost bounds shall be backwardly pro
portionall.

This is called the Troubled equation: In
Ordered equation the vpper ranke was tak
like vnto the neather, here it is taken vnlike
as 5 differeth from 7, so 4 from 6: and as 7
from 2, so 9 from 4: and as 2 from 8, so 3
from 9.

The proportion	5	7	2	8
of differences:	6	4	9	3

Therefore as 5 differeth from 8, so 3 from
6 doth differ.

The proportion	5	7	9	15
of reasons:	63	45	35	21

There

Therefore as 5 is to 15, so is 21 to 63. The bounds of Troubled proportion are commonly thus placed:

5 7 9 15

21 35 45 63

But this our way is as short and much more easie.

8 Continuall proportion is a proportion in which the same bound is taken for the second and third.

As in this example 1, 2, 3; as 1 differeth from 2, so 2 differeth from 3. Againe in this, 1, 5, 25: as 1 is to 5, so 5 is to 25.

9 If many middle bounds be continually taken for two, it is called Progression, in whose doctrine the first bound shall be alwaies the least.

This we do desire to be granted in the doctrine of Progression: although it may be otherwise.

CHAP. III.

Of Arithmeticall Proportion.

1 MOreouer Proportion is Arithmeticall or Geometricall. Arithmeticall, is the Proportion

Proportion of differences.

As 1 differeth from 2, so 4 differeth from 5.

2 The properties of disioyned Arithmeticall Proportion are two. The first is thus: if foure numbers be Arithmetically proportionall, the outmost bounds shall be equall to the middlemost.

As 16 differeth from 12, so 10 differeth from 6. Therefore the summe of ỳ outmost bounds 16 and 6, is equall to the summe of the middle bounds 12 and 10.

3 The second property is thus: if of twise two vnequall numbers giuen, the middle numbers shall be equall to the outmost, the foure assigned shall be Arithmetically proportionall.

The numbers assigned are 9, 7, 5, 3: the summe of the middle numbers 7 and 5, is equall to the summe of the outmost 9 and 3. Therefore the foure assigned are Arithmetically proportionall.

4 The properties of continuall Arithmeticall proportion are also two. The first is thus: If the Arithmeticall proportion be continuall with three bounds onely, the middle bound shall be halfe the summe of the outmost.

This is manifest by the 2 verse of this Chapter. As 3 differeth from 5, so let 5 differ from 7: I say that the summe of 3 and 7 is

double

double to 5. and so 5 to be the halfe of 3 + 7.

5 The second is thus: if the middle bound be the halfe of the summe of the outmost, it shall be the middle bound continually proportionall to the outmost.

This is apparent by the 3 verse of this Chapter. If 5 be the halfe of the summe of the two outmost 3 and 7, it shall be the middle bound Arithmetically proportionall vnto the.

CHAP. IIII.

Of Arithmeticall Progression.

1 THe doctrine of Arithmeticall Progression is of his differences and bounds: and in both places there is an inuention both of the seuerall and of the multitude. But diuersly, in the differences a double inuention of the multitude goeth before one seuerall inuention; in the bounds, a double inuention of the seuerall goeth before a double inuention of the multitude.

2 The inuention of the differences are first: If you take an vnity out of the number of the bounds, the remaine shall be the number of the differences.

Let the Progression of 6 bounds be this; 1, 3, 5, 7, 9, 11: I say that 6 — 1, that is, 5, is the number of the differences. The cause of this is, for that the middle bounds are continually taken for 2 by the 9 verse of the 1 Chapter.

3 If you take the first bound from the last, the remaine shall be the summe of the differences. As in the former example 1, 3, 5, 7, 9, 11: the first bound is 1, the last is 11: now 11 — 1 are 10, which is the summe of the differences.

4 If the first bound being taken from the last thou shalt deuide the remaine by the number of the bounds lesse by 1, the quotient shall be the differences of the Progression.

Of 8 children the eldest is 50 yeares of age, the youngest 27, and all of them from the first to the last do equally differ from other in age. I aske that difference. The outmost are 27, and 50, the number of the bounds 8. Now the quotient of 50 — 27, by 8 — 1, that is, of 23 by 7, is $3\frac{2}{7}$, the desired. Their ages then are thus: $27, 30\frac{2}{7}, 33\frac{4}{7}, 36\frac{6}{7}, 40\frac{1}{7}, 43\frac{3}{7}, 46\frac{5}{7}, 50.$

1 2 3 4 5 6 7 8

5 The inuention of the bounds followeth. If an vnitie being taken out of the number of the bounds, you shall multiply the remaine by the difference, and shall take the product from

from the last, the remaine shall be the first.

A traueller went from Barwicke to London in ten daies, in which he went euery day following 5 miles more then he did the day before. How many went he the first day, supposing that he went 48 miles the last day? I answer: 10 — 1, that is, 9 by 5, doth make 45, now 48 — 45, that is 3, is the desired.

Therefore

6 If an vnity being taken from the number of the bounds, you shall multiply the remaine by the difference, the product encreased by the first bound shall be the last.

As in the former example, suppose I knew not how many he went the last, but the first I know to be 3. I multiply 10 — 1, that is 9, by 5, and I make 45: then 45 + 3, that is, 48 is the last dayes iourney.

7 If the first bound being taken from the last thou shalt deuide the remaine by the difference, the quotient augmented by an vnity shal be the number of the bounds.

A man hath many children euery one of the is either two yeares older or yonger then other, the eldest is 48. the youngest is 16, how many children hath he? I answer: 48 — 16, that is, 32 deuided by 2, doth yeld the quotient 16: then 16 + 1 that is, 17 shall be the

number of the children.

8 If you shall multiply halfe the number of the bounds by the summe of the outmost, the product shall be the summe of the bounds of the Progression.

The father giueth his sonne on Newyeares day for a newyeares gift 12 d. which he doth euery day following for the whole moneth of January increase by 6 d. I desire the number of pence receiued. First, January hath 31 daies for the number of the bounds of the progression, the difference is 6, the first bound is 12, the last I know not: that I seeke by the 6 verse thus: 31 — 1. that is, 30 by 6, doth make 180. Now 180 + 12, that is, 192 is the last bound. Then halfe the number of the bounds is 15 $\frac{1}{2}$, and the summe of the outmost 12 and 192 is 204, the product of which by 15 $\frac{1}{2}$, that is 3,162 shall be the number of the pence receiued.

CHAP. V.

Of Geometricall Proportion, and of the disioyned kind.

1 THus farre of Arithmeticall proportion, Geometricall doth follow: which is the propor-

proportion of reasons; and it is commonly called by the generall name proportion.

Geometricall proportion hath for his excellencie deserued the name of proportion generally. Therefore by the name of proportion hereafter shall be vnderstood Geometricall proportion.

2 If the proportion be disioyned, commonly to the three bounds giuen, the fourth is sought: and of the three giuen, two are of the same kind betweene themselues.

As if 3 ℔ do yéeld me 5 s̄ commodity for a yeare, what shall 9 ℔ yéeld me? Here 3 ℔ and 9 ℔ are bounds of the same kind, for that they number things of the same kind.

3 If the second bound of the same kind greater, do require a greater bound, or a lesser do require a lesser, the proportion is outright: and the third bound is of the question.

When 15 ℔ are exacted of my neighbour, I pay 3 ℔: what shall I pay when 25 ℔ are exacted of him? Here the bounds of ỹ same kind are my neighbours 15 ℔ and 25 ℔, and when there is more exacted of my neighbour, there is also more demanded of me: therefore this proportion is outright, and the bounds be thus disposed: 15, 3, 25. Againe when I pay 25 s̄ my neighbour payeth 15 s̄: therefore

when I pay 5 s̄ what shall my neighbour pay? Here the bounds of the same kind are 25 and 5, then the second bound of the same kind lesser requireth a lesser bound: therefore the proportion is outright: and the bounds be thus disposed: 25, 15, 5. Such like questions are answered by ỹ 3 or 7 verse of the 6 Chapter of the 1 booke. I find therefore the fourth proportionall at the first question to be 5, at the second to be 3, so that when my neighbour payeth 15, I must pay 5, thus:

As 15 are to 3, so are 25 to 5.

And when I pay 25, my neighbour payeth 15, therefore when I pay 5, my neighbour shall pay 3.

A traueller hath 720 miles to go, whereof he goeth 36 in 2 dayes, in how many daies shall he go the whole iourney? I answer in 40.

A staffe of 6 foote long set plume vpon the plume ground doth yéld a shadow of 10 foote long: what is the height of that tower whose shadow at the same time is 125 foote long? I answer, 75 foote long.

If the $\frac{1}{6}$ part of the Moone be to our sight enlightened when she is $\frac{1}{12}$ part of a circle from the Sunne: how much shall she be enlightened when she is $\frac{1}{2}$ of a circle from the Sunne?

2

$2\frac{1}{8}$ els of cloth are bought for 27 shillings: what shall $5\frac{3}{4}$ els of the same cloth cost?

4 If the second bound of the same kind greater do require a lesser bound, or the lesser do require a greater, it is backward proportion: and the second bound is of the question.

When the bread weigheth $6\frac{2}{3}$ ounces, a bushell of wheate is sold for 3 s̄: when therefore a bushell of wheate is sold for 5 s̄, how many ounces must the bread weigh? Here the bounds of the same kind are the prices of the bushels, 3 s̄ and 5 s̄, and the greater price of wheate requireth the lesser weight of bread: therefore the proposition is backward, and the bounds be thus disposed.

$6\frac{2}{3}$ 5 3

Againe when the bread is of 4 ounces, the bushell of wheate is sold for 5 s̄: If therefore the bushell be sold for 3 s̄, what shall ỹ weight of bread be? Here the lesser price of wheate requireth a greater weight of bread: the bounds be thus disposed 4, 3, 5. In such like questions the fourth proportionall is found by the 4 or 11 verse of the 5 Chapter of the 1 booke. I find therefore at the first question the fourth proportionall to be 4, thus: $6\frac{2}{3}$, 5, 3, 4. In the second to be $6\frac{2}{3}$ thus: 4, 3, 5, $6\frac{2}{3}$.

Three mils in 2 dayes do grind 20 bushels

of wheate, in how many dayes shall 5 mils grind so much in the same time? The propoztion is thus: 3 5 20 $1\frac{1}{3}$.

A cut-throate vsurer desireth to know in what time the loane of 112 ℔ shall be equall to the loane of 66 ℔ for 7 moneths. Answer, 66, 112, 7, $4\frac{1}{2}$.

A messenger or lackey goeth a iourney of 600 miles in 24 dayes when the day is 12 houres long: in how many dayes, shall he go the same when the day is 16 houres long? I answer, in 18 dayes.

5 If to the assigned bounds of the same kind there be equall numbers adioyned, the equall neglected, the proportion shall be made.

If two oxen do are 4 akers of land in $23\frac{1}{3}$ dayes, in how many dayes shall 2 oxen are 3 akers? This outright propoztion is thus concluded: 4 $23\frac{1}{3}$ 3 $17\frac{1}{2}$.

If two oxen are 3 akers of land in $17\frac{1}{2}$ dayes, in how many dayes shall 7 oxen are 3 akers? This backward propoztion is thus concluded: 2 7 $17\frac{1}{2}$ 5.

CHAP.

CHAP. VI.

Of the common disioyned Proportion and of the examples of his simple Proportion.

1 DIsioyned Proportion is Common or Artificiall. The Common Proportion is manifest out of that which hath bene said. In it the examples are sometime of simple Proportion, sometime of Manifold. The examples following are of simple Proportion.

A merchant of Strawsborow in Germany buieth 100 ℔ worth of spices at Frankford: the weight of spices is 800 pounds, the tolle was 6 ℔, the carriage cost 9 ℔, the charges of the factor 7 ℔: he desireth to gaine 40 ℔: for how much therfore must he sell one pound of spice? I answer for $\frac{8'}{400}$ ℔, that is by the 3 verse of the 17 Chapter of the 1 booke, 4 s̄, ob,q, or thereabout. Here Addition went before Proportion, in the examples following the Proportion shall follow Subduction.

Two merchants bought a peece of cloth for 10 ℔, and sold it for 12 ℔. If they had sold it for 14 ℔, what should they haue gained? I subduct the one from the other, and I find the gaine of 10 ℔ to be 2 ℔. Then I say, 10 giue 2, therefore

fore 14 shall giue $2\frac{4}{7}$ for the gaine desired.

$\frac{1}{4}$ of a Tower is hidden in the earth, $\frac{1}{6}$ vnder the water, the other part being 120 foote is aboue the water: therefore how many foote were within the earth? how many within the water? and how long was the whole tower? First $\frac{1}{4}+\frac{1}{6}$, is $\frac{5}{12}$: then $1-\frac{5}{12}$ are $\frac{7}{12}$. Now $\frac{7}{12}$ are 120: therefore $\frac{1}{4}$ shall be $51\frac{3}{7}$, and $\frac{1}{6}$ shall be $34\frac{2}{7}$. So that the whole length shall be $205\frac{5}{7}$ foote.

Sometime the two first bounds of the Proportion are contained in one: as, what number containeth in it 12 thrise and $\frac{2}{3}$? How many twelfs are there in $\frac{3}{4}$? Here $3\frac{2}{3}$, and againe $\frac{3}{4}$ do containe the two first bounds of the question: but for $3\frac{2}{3}$ we take $\frac{11}{3}$, by which is signified, that as oft as 3, are contained in a 11, so oft 12 are contained in the number sought, the number sought is 44, the Proportion is thus:

3 11 12 44

In the other question the Proportion is thus: 4 3 12 9. The former examples were of outright Proportion: those which follow shall be of backward Proportion.

15 oxen do are a field in 8 dayes, in how many dayes shall 20 are the same? The Proportion is thus: 8 20 15 6.

Three thousand are beseeged which cannot

loke for aide vntill 9 moneths be ended: their victuals will last them but for 3 moneths: how many of vs therefore must be kept within the Citie? 3000 9 3 1000.

CHAP. VII.

Of the examples of Manifold Proportion.

A Workman toke a well of 34 yards deepe, to digge for 60 l, hauing digged 20 yards he fell sicke, and so requireth his wages due, how much therefore is it? At the first blush it would seeme to be thus concluded: 34, cost 60: therefore 20 must cost $35\frac{5}{17}$. But the second yard containeth ỹ labour of two yards, the third of three, and so forth those which follow containe their owne labour and the labor of all the former. Wherefore two Arithmeticall progressions do go before the conclusion of the question. The first is of 34 bounds, and his outmost are 1, and 34 the summe of the bounds by the 8 verse of the 4 Chapter, is 593. The second progressiõ is of 20 bounds, whose outmost are 1 and 20: the summe of ỹ bounds by the 8 verse of the 4 Chapter is 210. Now

as

as 595 yards are to 60 ℔, so are 210 yards to 21 $\frac{3}{17}$ ℔, that is, to 21 ℔ 3 ß 6 ð and $\frac{6}{17}$ ð.

Of two lackeys the first maketh account to go from Yorke to London in 5 dayes; the other from London to Yorke in three dayes: they both take their iourney to day at 12 of the clocke: when and where shall they méete? Here two proportions do go before the question. The first is thus: The first lackey will go the iourney in 5 dayes, therefore in 1 day he will go $\frac{1}{5}$ of the iourney. The second is thus: The second in 3 dayes will go the iourney, therefore in one day he shall go $\frac{1}{3}$ of the same. Now $\frac{1}{5}+\frac{1}{3}$, that is, $\frac{8}{15}$, is the first bound of the chiefe proportion: it is thus concluded, $\frac{8}{15}$ of the iourney are gone in one day, therefore $\frac{1}{1}$ of the iourney, that is, the whole is gone in 1 $\frac{7}{8}$ of a day. Therefore they shall méete two dayes hence (this being tuesday in the equinoctiall) that is, vpon thursday at $\frac{1}{2}$ houre after 10 of the clocke. It remaineth where they shall méete. The first doth go the iourney in 5 dayes, therefore in 1 $\frac{7}{8}$ he shall finish $\frac{3}{8}$ of the iourney. Wherefore they shall méete at $\frac{3}{8}$ of the iourney finished by the first, or at $\frac{5}{8}$ finished by the second, that is, betweene Tuxford and Newmarke.

A conduit hath three cockes: if they shall se-

uerally

uerally runne, the first will fill the cesterne vnderneath it in $\frac{1}{4}$ of an houre, the second in $\frac{1}{2}$, ỹ third in 1 houre. If they shal all ioyntly run, in what space will they fill the cesterne? Here two proportions do go before the question. The first is thus: $\frac{1}{4}$ of an houre filleth it once; therefore in 1 houre it shall fill it 4 times. The second is like it: $\frac{1}{2}$ houre filleth it once, therefore 1 houre filleth it twice. Now $\frac{1}{4} + \frac{1}{2} + 1$, ỹ is 7, is the first bound of the chiefe proportion, signifying that all the cockes being let runne they will fill the cesterne 7 times in an houre. Therefore I conclude; if the cesterne be filled 7 times in 1 houre, that it shall be once filled in $\frac{1}{7}$ of an houre.

Of 4 Carpenters, the first could build the house in one yeare, the second in 2, the third in 3, the fourth in 4. In what space would all of them finish the same? Answer, if the second could do it in 2 yeares, the third in 3, ỹ fourth: then the second in 1 yeare shall do $\frac{1}{2}$ of the worke, the third $\frac{1}{3}$ the fourth $\frac{1}{4}$. Now $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$, that is, $\frac{25}{12}$ is the first bound. Therefore if $\frac{25}{12}$ of the worke is done in 1 yeare, then ỹ whole shall be done in $\frac{12}{25}$ of one yeare. That is, in 5 moneths, 22 dayes, 19 houres, and $\frac{1}{5}$ of an houre.

A Fountaine hauing 2 pipes or cockes hath

a

a cesterne betweene them, which if they shall severally runne the vpper pipe will fill the cesterne in 4 houres, the other will empty it in 11 houres. If they shall both runne, in what space will they fill it? Here two proportions do go before the chiefe proportion. The first is thus: If the vpper pipe shall fill it in 4 houres, then in 1 houre it shall fill $\frac{1}{4}$ of it. The second is thus: If the neather pipe shall empty it in an 11 houres, then in 1 houre it shall empty one $\frac{1}{11}$ part of it. Now $\frac{1}{4} - \frac{1}{11}$ is $\frac{7}{44}$. Therefore I conclude $\frac{7}{44}$ are filled in 1 houre; therefore the whole shall be filled in $6\frac{2}{7}$ of an houre. In the examples following Addition goeth before Subduction.

Of three Carpenters, the first can finish the house in 30 dayes, the second in 40, the third if he be ioyned with the other two, can do it in 15 dayes. In what time would he do it alone? Here two proportions do go before the question. The first is thus: The first can do the worke in 30 dayes, therefore in 15 dayes he shall do $\frac{1}{2}$ of the worke. The second is thus: the second can do the worke in 40 dayes, therefore in 15 dayes he shall do $\frac{3}{8}$ of the same. Now $\frac{1}{2} + \frac{3}{8}$ are $\frac{7}{8}$, the $\frac{8}{8} - \frac{7}{8}$, is $\frac{1}{8}$ for the first bound. If the third shall do $\frac{1}{7}$ of the worke in 15 daies; he shall do the whole in 120 daies.

CHAP.

CHAP. VIII.

Of the disioyned artificiall Proportion, and of the Addition and Subduction of proportions and proportionals.

1 DIsioyned artificiall Proportion is that which moreouer and beside the aboue-said hath some especiall cunnings. It is Compounded or Continued. Compounded is that which containeth the third Proportion found by the Numeration of two proportions of some certaine kind: and it is first Addition or Subduction, then Multiplication.

There may be also Diuision of proportions; for if the Proportion 3, 4, 17 1/2, 23 1/3, multiplied by the Proportion 2, 7, 5, 17 1/2, do make the Proportion 3, 14, 5, 23 1/3. It followeth that 3, 14, 5, 23 1/3, deuided by 2, 7, 15, 17 1/2, shall yéeld in the quotient the Proportion 3, 4, 17 1/2, 23 1/3. But I know no vse of this Diuision.

2 Addition addeth two proportions of equall consequents.

That is to wit, here the first reasons of the proportions, and againe the second are added betweene themselues. The rule of this Addition is thus:

3 If the followers of two proportions assigned be equall, as the summe of the first formers

mers is to the first follower, so the summe of the second formers is to the second follower.

As 24 is to 6, so is 36 to 9; and as 2 is to 6, so is 3 to 9: I say that as 24 + 2 is to 6, so is 36 + 3 vnto 9.

24		2	24
6		6	6

In like manner the reason of 36 to 9, added to ỹ reason of 3 to 9, is the reason of 36+3 vnto 9.

36		3	39
9		9	9

Wherefore as 24+2 is to 6, so is 36+3 vnto 9, by the 7 verse of the 3 Chapter of the first booke.

Therefore

4 If foure numbers be proportionall, as the first and second are to the second, so the third and fourth are to the fourth.

This is a certaine abridgement of the former addition: for if you shall repeate the second and fourth bounds of the proportion assigned, you shall make another proportion of equall followers to the assigned. As 3 is to 4, so is 6 to 8: here you may say, as 4 is to 4, so is 8 vnto 8: and therefore by the 3 verse, as 3+4 is vnto 4, so is 6+8 vnto 8.

The addition of many proportionall bounds ari-

arising out of the next former additions, is thus.

5 If many numbers assigned be proportionall, as all the formers are to all the followers, so one of the formers shall be vnto his follower.

As 3 is to 6, so is 4 to 8, and so is 5 to 10, and so is 7 to 14: I say as 3+4+5+7: is to 6+8+10+14: so is 3 to 6. Here by the way you shal marke that such a like Addition is not an addition of reasons but of bounds. For the addition of reasons followeth the addition of parts, which numbreth the Numerators onely; so this doth number the formers onely of the same consequents: but here aswell the formers were added betwéne themselues, as the followers betwéne themselues.

6 Subduction having two proportions assigned of equall followers, taketh the one out of the other.

That is to say, here the first reason of the second proportion, and againe the second reason of the second proportion is taken out of the second reason of the first proportion. The rule of this Subduction is thus.

7 If the following bounds of two proportions assigned be equall, it shall be as the difference of the first formers is to the first follo-

wer, so the difference of the second formers is to the second follower.

As 26 is to 6, so is 39 to 9: and as 24 is to 6, so let 36 be to 9: I say as 26—24 is to 6, so is 39—36 to 9.

26	24	2
6	6	6

And in like manner the reason of 36 to 6, being taken out of the reason of 39 to 9, doth leaue the reason of 39—36 to 9.

39	36	3
9	9	9

Wherefore by the 9 verse of the 3 Chapter of the first booke, as 26—24 is to 6, so is 39—36 vnto 9. Therefore

8 If foure numbers be proportionall, as the first diminished by the second, is vnto the second, so the third diminished by the fourth, shal be vnto the fourth.

This is a certaine abridgement of the former Subduction: as 7 is to 4, so is 14 to 8: here you may say as 4 is to 4, so is 8 vnto 8, and therefore by the 7 verse, as 7—4 is to 4, so is 14—8 vnto 8. The addition of proportionall bounds arising out of the former verse, is thus.

9 If as the whole is to the whole, so the subducted shall be vnto the subducted; it shal be as the

the whole is to the whole, ſo the remaine to be to the remaine.

As 3+4+5+7, is to 6+8+10+14: ſo let 3 be to 6: I ſay as 3+4+5+7, is to 6+8+10+14, ſo 4+5+7 is to 8+10+17. Here alſo thou ſhalt marke as before ſuch a like Subduction to be of bounds and not of reaſons, becauſe the Subduction of reaſons following the Subduction of parts doth not number the follower, yet this ſubduction doth number as well the followers betweene themſelues, as the formers betweene themſelues.

CHAP. IX.

Of the rule of fellowſhip, and his firſt kinde.

1 ADdition of bounds for his ſingular vſe in deciding of controuerſies, is commonly called the Rule of fellowſhip.

Here the ſumme of the followers of the latter reaſons, is both giuen for the finding out of the followers ſeuerally, and is taken for the follower of the firſt reaſon: for the former of the ſame reaſon we take the ſumme of the former of the latter reaſons.

2 The Rule of fellowship is twofold: the first is when the formers of the latter reasons are seuerally giuen.

Two men being parteners, the first brought 8 ℔, the second 6 ℔, they haue gained 7 ℔, what is each mans part? Here the assigned formers of the latter reasons are 8 and 6, whose summe is 14: the question is thus answered.

14 do gaine 7, therefore $\left\{\begin{matrix}8\\6\end{matrix}\right\}$ haue gained $\left\{\begin{matrix}4\\3\end{matrix}\right\}$

Thrée part 100 ℔ thus, that when the first shall take $\frac{1}{3}$ and $\frac{1}{4}$, the second shall take $\frac{1}{4}$ and $\frac{1}{5}$, the third $\frac{1}{5}$ and $\frac{1}{6}$. This may séeme at the first blush or view to be absurd: for if the first for $\frac{1}{3}$ and $\frac{1}{4}$ shall take $33\frac{1}{3}$ ℔, and 35 ℔, that the second cannot haue his whole due, and the third shall haue right nought. Yet notwithstanding the Rule of fellowship shall absolue ẏ question without any absurdity. For here the summe of ẏ seuerall formers $\frac{35}{60}$, $\frac{27}{60}$, and $\frac{22}{60}$, is $\frac{84}{60}$: wherefore the parts being reduced vnto whole numbers by the Commentarie of the 5 verse of the 18 Chapter, of the first booke we conclude the question thus:

84 are taken for $\left\{\begin{matrix}35\\27\\22\end{matrix}\right\}$ are taken for $\left\{\begin{matrix}41\frac{2}{3}\\32\frac{1}{3}\\26\frac{4}{21}\end{matrix}\right\}$ an 100, therefore

Foure do so deuide 600 ℔, that when the first

first shall receiue $\frac{3}{5}$ and 9 l, the second shall receiue $\frac{3}{4}$ and 8 l, the third $\frac{5}{6}$ and 7 l, the fourth $\frac{7}{8}$ and 6 l. Here also the two first parts are more then the whole, so that this kind of example may seeme as absurd as the former: but the Rule of fellowship shal end this controuersie thus: First the summe of 9. 8, 7, and 6, is 30, then 600 — 30, is 570, for the first reasons follower: againe the parts $\frac{2}{5}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$, are reduced by the Commentarie of the 5 verse of the 18 Chapter of the first booke, vnto whole numbers, 80, 72, 100, and 105. The summe therefore of the reduced numbers for the former of the first reason is 357, the question is thus answered.

357 are taken for { 80, 72, 100, 105 } are taken for { $127\frac{261}{357}$, $114\frac{114}{119}$, $159\frac{79}{119}$, $167\frac{11}{17}$ }
570, therefore

But moreouer 127 + 9, is 136. Wherefore the part of the first shall be $136\frac{261}{357}$: againe 114 + 8, are 122: wherefore the seconds part shall be $122\frac{114}{119}$. In like manner 159 + 7, are 166: wherefore the part of the third shall be $165\frac{79}{119}$. Lastly, 167 + 6, are 173, wherefore the fourth mans part shall be $173\frac{11}{17}$.

CHAP. X.

Of the second rule of fellowship.

1 THe second rule of fellowship is that which findeth the seueral formers of the latter reasons: and that by Multiplication of assigned, or by Addition of the products of the assigned; there the products are to the followers as the summe of the products is to the summe of the followers: here the summe of the products are to the followers as the summe of them is to the summe of the followers.

An example where the seuerall formers are found by Multiplication of the assigned. A legion of souldiers containeth 6,100 footmen, and 726 horsemen; and the wages of the footemen is 4 ℔ for one moneth, the wages of the horsemen 9 ℔. A bootie of 2000 ℔ is to be parted amongst them; how much must the footemen haue, how much the horsemen? Here the product of the wages of the footemen by their number is 24,400, and the product of the wages of the horsemen by their number is 6,534: which products are the formers of the latter reasons, and the summe of them is 30,934: therefore I conclude the question thus:

30,934, do giue $\left\{\begin{matrix}24,400\\6,534\end{matrix}\right\}$ shall $\left\{\begin{matrix}1,577\frac{8,541}{15,467}\\422\frac{6,926}{15,467}\end{matrix}\right\}$ gaine 2,000, therefore

An

An example where the seuerall formers are found by Addition of the products of the assigned. Foure merchants being partners for two yeares, the first gaue 30 l. to the stocke; but afterward, 8 moneths being expired, he toke away 10 l. and againe in the beginning of the twentieth moneth he brought 12 l. againe. The second in the beginning brought 24 l. and after 6 moneths expired, he withdrew 8 l: and then againe in the beginning of the 16 moneth, he brought againe 14 l. The third at the beginning brought 20 l. and the 7 moneth ended, he drew backe his whole stocke; but then in the beginning of the 18 moneth he restored 16 l. The fourth in the entrance of the 7 moneth laid in 18 l. but after 4 moneths ended he withdrew 9 l; and againe at the beginning of the 10 moneth he added 15 l. They gained 100 l. The gaine of euery man is demanded. Here the moneths being multiplied by each mans charges, the summes of ỹ same products shall be taken for the formers of the latter reasons, and the question shall be thus concluded. First therefore the products of 30 by 8, of 20 by 11, of 32 by 5, are 240, 220, and 160, the summe of them is 620; for the first mans charge. Then the products of 24 by 6, of 16 by 9, of 30 by 9, are 144, 144 and 270: the

summe of them is 558: for the second mans charge. Thirdly, the products of 20 by 7, of 16 by 7, are 140 and 112; the summe of them is 252, for the third mans charge. Fourthly, the products of 18 by 4, of 9 by 6, and of 24 by 8, are 72, 54, and 192, and their summe is 318: for the charge of the fourth partener. Now the summe of the charges of euery man 620, 558, and 318, is 1,748. Therefore the desired gaine of each man I thus conclude:

1,748 do gaine	620	shall gaine	$35\frac{205}{437}$ l
100 l, therefore	558		$31\frac{403}{437}$
	252		$14\frac{182}{437}$
	318		$18\frac{234}{437}$

CHAP. XI.

Of the Multiplication of Proportions.

1 MVltiplication multiplieth two Proportions, whereof the first hath the third bound equall to the fourth of the second. Here

2 If the Proportions multiplied be outright, it shal be as the product of the first bound is to the product of the second, so the second bound of the second Proportion shall be to the fourth of the first.

As

As 3 is to 4, so let $17\frac{1}{2}$ be to $23\frac{1}{3}$: and as 2 are to 7, so let 5 be to $17\frac{1}{2}$. Againe the product of the reason of 3 to 7, by the reason of 2 to 7. let it be the reason of 3 to 14: I say as 3 is to 14, so is 5 to $23\frac{1}{3}$.

3 If the Proportions multiplyed be backward Proportions, it shall be as the product of the former bounds is to the product of the latter bounds, so the fourth bound of the first Proportion shall be to the third of the second.

This shall be manifest in the same Proportions if you shall make the second Proportion of the assigned, the first: and in both of them shalt put the third bound in the fourth place thus.

2	7	$17\frac{1}{2}$	5
3	4	$23\frac{1}{3}$	$17\frac{1}{2}$
3	14	$23\frac{1}{3}$	5

The vse of these multiplications is most singular in the Proportion which is commonly called Double of double proportions of the same kind assigned. For in it the fourth bound to the three assigned of the proportion made, is sought: but of the three assigned one only indeede is giuen, the other two are giuen in virtue. In the double outright proportion, twise two bounds of the same reason of the former reasons, are giuen for two products of them.

In

In the double backward twise two bounds of a diuerse reason of the former reasons are giuen, for the products of the bounds of the reasons of the same kind: in the double outright the product of the two first, in the double backward the product of ỹ bounds of the same reason, the first and fift is the first bound of the proportion made: lastly there the bounds of the proportion made are altered, here they change not their place.

Examples of double outright proportion.

1 Three pounds in 2 moneths do gaine 5 ℔, therefore what shall 4 ℔ gaine in 7 moneths: Here the two first reasons of 3 to 4, and of 2 to 7, multiplied betweene themselues, do make the reason of 3 to 14, thus:

	1	
3	2	3
4	7	14
2		

And then as 3 is to 14, so is 5 to $23\frac{1}{3}$. Wherefore if 3 ℔ in 2 moneths do gaine 5 ℔, then 4 ℔ in 7 moneths shall gaine $23\frac{1}{3}$ ℔.

2 When a bushell of wheate is at 15 ß price, the bread of 8 ounces is sold for 2 ß, therefore when a bushell of wheate shal be of 20 ß price, what shall the price of the bread be of 15 ounces: Here the reasons of 15 to 20, and of 8 to

15,

15, multiplied betwæne themselues, do make the reason of 2 to 5, thus:

1	2	2
~~15~~	8	
~~20~~	~~15~~	5
5	1	

Now as 2 is to 2, so is 5 to 5. Wherefore when the bushell shall be of 20 d price, ỹ bread of 15 shall be sold for 5 d.

3 An 100 l weight are carried 20 miles for 36 l, therefore 50 l weight for how many l, shall they be carried 12 miles? Here the reasons of 100 to 50, and of 20 to 12, being multiplied betwæne themselues, do make the reason of 10 to 3, thus:

2	5	10
~~100~~	~~20~~	
~~50~~	~~12~~	3
1	3	

Now as 10 is to 36, so is 3 to $10\frac{4}{5}$. Therefore 50 l, weight shall be carried 12 miles for $10\frac{4}{5}$ l.

4 Fiftæne elles of cloth $\frac{3}{4}$ broade, are sold for 6 l: how much shall 40 elles $1\frac{2}{3}$ broade be sold for? Here the reasons of 15 to 40, and of $\frac{3}{4}$ to $\frac{5}{3}$: multiplied betwæne themselues shall make the reason of $\frac{9}{4}$ to $\frac{40}{3}$, thus:

3	3	9
15	4	4
40	5	40
8	3	3

Now as $\frac{9}{4}$ are to 6, ſo $\frac{40}{3}$ are to $35\frac{5}{9}$. Therefore 40 elles of cloth of one elle broade ſhall be ſold for $35\frac{5}{9}$ l. Examples of double backward Proportion. Two oxen do are 4 akers of land in $23\frac{1}{3}$ daies, in how many daies ſhall 7 oxen are 3 akers? Here the aſſigned reaſons of 2 to 7, and of 3 to 4, multiplied betweene themſelues, do make the reaſon of 3 to 14 thus:

1		
2	3	3
7	4	
	2	14

Now as 3 is to 14, ſo is 5 to $23\frac{1}{3}$. Therefore 7 oxen ſhall are 3 akers in 5 daies.

In 2 moneths 7 l do gaine 3 l, in how many moneths ſhall 4 l gaine 3 l? Here the reaſons of 7 to 4, and of 3 to 3, multiplied betweene themſelues do make the reaſon of 7 to 4 thus:

	1	
7	3	7
4	3	4
	1	

Now as 7 is to 4, ſo is $2\frac{1}{3}$ to 2. Therefore

$3\frac{1}{3}$

3 ½ moneths shall gaine 3 £ of 4 £.

Two firkins of béere do serue 30 persons for 3 daies, therefore three firkins how many shall they serue for 4 daies? Here the reasons giuen of 2 to 3, and of 4 to 3, multiplied betwéene themselues do make the reason of 8 to 9 thus:

2	4	8
3	3	9

Now as 8 is to 9, so is 26⅔ to 30. Therefore here 3 firkins shall serue 26 persons for 4 daies, and yet there doth remaine ⅓ of that béere which some of them might haue drunke in 4 daies.

CHAP. XII.

Of the Continued Proportion.

1 THus farre of the Compounded Proportion. The Continued now followeth which taketh the simple bounds betweene themselues of the assigned seuerall reasons, and they placed in the vpper-ranke vnderneath them setteth the least continually Proportional found.

As in this example:

2	3	4	5	6	7
	16	24		30	35

Here

Here the the assigned reasons are 2 to 3, and 4 to 5, and 6 to 7, and the continually Proportionalls found are 16, 24, 30, 35. The expressing of the continued Proportion is thus: as 2 is to 3, so is 16 to 24: and as 4 is to 5, so is 24 to 30: and as 6 is to 7, so is 30 to 35. Therefore here the bounds are continued not in the same Proportion, but in a diuerse: which I do therefore aduertise lest any man should take the continued Proportion for the continuall.

2 The finding of the least continually Proportionall in the assigned reasons is thus: If of the least Proportionals to the assigned second and third, the second shall multiply the bounds of the first reason, the first the second bound of the second reason, the products shall be the least continually proportionall to the foure assigned.

Let the reasons giuen in the least bounds be the reason of 2 to 9, and of 15 to 4, and the least proportionals vnto 9 and 15, let them be 3 and 5: and then the products of 2 and 9 by 5, let them be 10 and 45, and the product of 4 by 3 let it be 12.

2	9	15	4
	3	5	
10	45	12	

I say, as 2 is to 9, so 10 is to 45, and as 15 is

is to 4, so is 45 to 12.

A man making his will, his wife being great with child, thus bequeatheth his goods: If she shall beare me a sonne, he shall inherite $\frac{2}{3}$ of my goods; and my wife shall inherite the rest: If she shall bring forth a daughter, she shall haue $\frac{1}{3}$ of my goods, and my wife shall haue the rest: Now the man being dead there were borne to him by his wife two sonnes and one daughter: the question is what shall be ỹ portion of those goods due to the mother, and what the portion due to each of the children according to the will of the deceassed father. Here the least continually proportionall vnto the assigned reasons of 2 to 1, and of 2 to 1, by the former verse are 4, 2, and 1.

2	1	2	1
4		2	1

By which is signified each of the sonnes shall take 4, then the mother is to take 2, and the daughter 1. If therefore the goods being praised he shall be found to haue bene worth 1,560 l, the parts of the heires shall be the concluded by the rule of fellowship thus.

11 do receiue 1,560 l, therefore $\left\{\begin{matrix}4\\2\\1\end{matrix}\right\}$ shall receiue $\left\{\begin{matrix}567\frac{3}{11}\text{ l}\\283\frac{7}{11}\\141\frac{9}{11}\end{matrix}\right.$

So that the part of each of the sonnes shall be

be 367 $\frac{3}{11}$ ℔, the part of the mother 283 $\frac{7}{11}$ ℔, the part of the daughter shall be 141 $\frac{9}{11}$ ℔.

3 Againe if of the least proportionals to the third product and to the former bound of the third reason, the second shall multiply the products now made, and the first shall multiply the follower of the third reason, the products shall be the least continually proportionall to the fixe assigned: and so foorth of many more infinitely.

As 2 is to 9, so is 10 to 45, and as 15 is to 4, so is 45 to 12. The reasons giuen were the reason of 2 to 9 and the reason of 15 to 4: now let vs make them to be 2 to 9, 15 to 4, and 18 to 29. Then the least proportionall to the assigned 12 and 18, let them be 2 and 3: againe the products of 3 by 12, 45 and 10, let them be 36, 135, and 30, and the product of 2 by 29, let it be 58.

2	9 |	15	4 |	18	29|
10	45		12		
			2	3	
30		135	36		58

I say as 2 is to 9, so 30 is to 135, & as 15 is to 4, so 135 is to 36, and as 18 is to 29, so 36 to 58.

To 4 executors 500 ℔ are bequeathed, that when the first shall take 2, the second shall take 4, when the second shall take 6, the third shall take

take 8; when the third shall take 10, the fourth shall take 12: I would know each mans part. Answer: Here by the first verse of this Chapter, for 2 and 4, 6 and 8, 10 and 12. I take 1, 2, 3, 4, 5, 6. Now by the next verse, the least continually proportionall to the assigned are 15, 30, 40, 48: by which is signified when the first shall take 15, the second shall take 30, the third 40, the fourth 48. And the summe of these parts is 133: wherefore by the first rule of fellowship, the parts of the executors I thus conclude.

133 do give 500 l.	15	shall	56 $\frac{56}{133}$
	30		112 $\frac{104}{133}$
Therefore	40	give	150 $\frac{10}{133}$
	48		180 $\frac{60}{133}$

CHAP. XIII.

Of continuall Proportion in three bounds onely.

1 THe doctrine of continuall Proportion is considered, sometime in three bounds onely, sometime in more. In three bounds onely, first there is a finding both of the meane and third bounds continually proportionall, then a certaine property of whole numbers of many.

2 The finding of the meane proportionall is thus: If the third shall deuide the product of two by it selfe, the Diuisor shall be the meane proportionall to those two.

The product of 4 by 9, let it be 36: which let 6 deuide by it selfe, that is, by 6. Here by the 10 verse of the 6 Chapter of the first booke, as 4 is to 6, so is 6 to 9.

3 The finding of the third proportionall is twofold: The first is thus: If the first shall deuide the product of the second, the quotient shall be the third proportionall to them.

Let the assigned be 6 and 10, and the product of 10 deuided by 6, let it be $16\frac{2}{3}$, here by the 7 verse of the 6 Chapter of the first booke, as 6 is to 10, so is 10 to $16\frac{2}{3}$.

4 The second is thus: If the second shall multiply the quotient of the second deuided by the first, the product shall be the third proportionall to the assigned.

Let the assigned be 4 and 8, and then let the quotient of the second 8 deuided by 4, the first be 2, and the second multiplying the quotient let it make 16: here by the 3 verse of the 6 Chapter of the first booke, as 4 is to 8, so is 8 to 16.

CHAP.

CHAP. XIIII.

Of the reason of two simple numbers vnto the reason of the outmost, and of the finding of the continually Proportionals.

1 IN many continually Proportionall is considered, first the reason of two simple numbers vnto the reason of the outmost: Then the finding of the continually Proportionals, and of the summe of the bounds.

2 The comparison of reasons goeth before: If you take an vnity out of the number of the bounds, the reason of the first bound vnto the second multiplyed, according to the denomination of the remaine, shall be the reason of the first vnto the last.

Let the progression of 5 bounds be 2, 4, 8, 16, 32, and 5 — 1 be 4. here by the Commentarie of the 7 verse of the 7 Chapter of this booke, and by the 4 verse of the 18 Chapter of the first booke, the reason of 2 to 4, being foure folded, is the reason of 2 to 32. The worke is thus performed by the 4 verse of the 18 Chapter of the 1 booke.

	1	1	1	
2	4	8	XVI	2
4	8	XVI	32	32
1	1	1		

To double, to threefold, to fourefold a reason, is not to adde the same reason twise, thrise or foure times set downe, as the common sort do thinke; but to multiply it continually by it selfe being set downe twise, thrise, foure times: whether the bounds be the same, or diuerse of the same reason, it is no matter.

3 The finding of the continually proportionalls doth either consider the assigned progression, or doth find the desired. There one bound onely is found in the progression assigned, here many in the reason assigned.

4 The finding of the continually proportionall in the progression assigned is thus: If the first bound of an infinite progression shall deuide some one of the followers, the product of the quotient by any of the assigned, shall be a bound of the assigned progression.

Let 3, 6, 12, 24, 48, be an infinite progression, and then let 3 deuide 12 by 4, the which 4 let it be multiplyed by which of the assigned you please: I say, the product is a bound of the assigned progression. Therefore

5 If to the bounds of the assigned progression their numbers shall answer orderly, if the quotient shall multiply the deuided, the double of the number answering to the multiplied lesse by one, shall be the number of the

product.

product. As here

1	2	3	4	5	6	7	1	2	3	4	5	6
1	2	4	8	16	32	64	3	6	12	24	48	96
			$\frac{8}{64}$							$\frac{4}{48}$		

6. But if the quotient ſhall multiply another then the deuided, the ſumme of the numbers anſwering to the deuided, and multiplied leſſe by one, ſhal be the number of the product. As here.

1	2	3	4	5	6	7	1	2	3	4	5	6	
1	2	4	8	16	32	64	3	6	12	24	48	96	
		4		$\frac{4}{64}$					4		$\frac{4}{96}$		

The vſe of theſe two verſes is moſt ſinguler in Algebraicall Multiplication and Diuiſion.

7. The finding of the continually proportionalls in the aſſigned reaſon, doth take the bounds of the aſſigned reaſon ſimple numbers of many betweene themſelues.

This foundation layed,

8 If the former of the reaſon aſſigned ſhall multiply it ſelfe and the follower, and the follower himſelfe onely, the three products ſhall be the leaſt continually proportionall in the aſſigned reaſon.

Let the bounds of the aſſigned reaſon of 2 to 3, be ſimple betweene themſelues: and the products of 2 by 2, and 3, let them be 4 and 6;

and againe the product of 3 by it selfe let it be 9.

2 3

4 6 9

Here by the 5 verse of the 6 Chapter of the first booke, as 2 is to 3, so is 4 to 6, and so is 6 to 9. Then 4, 6, and 9, are simple betwéene themselues, because 4 and 9 are simple betwéene themselues, by the 6 verse of the 14 Chapter of the first booke.

9 Then if the products shall be multiplied by the assigned former, and againe the last by the assigned follower, the foure products shall be the least continually proportionals in the assigned reason, and so in more.

Let the former examples be repeated.

2 3

4 6 9

And then let the products of 2 by 4, 6 and 9, be 8, 12, and 18. and then againe let the product of 9 by 3 be 27.

2 3

4 6 9

8 12 18 27

CHAP.

CHAP. XV.

Of the finding of the ſumme of the bounds continually Proportionall.

1 THe finding of the bounds continually proportionall doth follow: If the firſt bound being taken out of the ſecond and laſt, it ſhal be as the remaine of the ſecond is to the firſt, ſo the remaine of the laſt be vnto the number found, the ſumme of the number found and of the laſt ſhall be the ſumme of the bounds of the aſſigned progreſſion.

As 32 is to 48, ſo let 48 be to 72, and ſo let 72 be to 108: Againe let 48 — 32, & 108 — 32, be 16 and 76.

	16		76
32	48	72	108
	32		32

Againe as 16 is to 32, ſo by the 3 verſe of the 6 Chapter of the 1 booke, let 76 be to 152.

16	32	76	152

I ſay 152 + 108, is the ſumme of 32, 48, 72, and 108. The father giueth his ſonne on the firſt day of the yeare for a newyeares gift 1 d, on the ſecond day 3 d, on the third day 9 d, and euery day following for the whole moneth of January he threefoldeth vnto his ſonne the gift of the former day: I would know what

the summe of the gifts shall be? Here first the moneth of January hath 31 dayes for ỳ number of the bounds of the progression assigned.

1 2 3 4 5 6 7 8 9
1 3 9 27 81 243 729 2,187 6,561

Then the summe of ỳ numbers 8 and 9 is 17, and the bounds answering to them are 2,187 and 6,561 the product of which is 14,348,907, for the sixteenth bound of the progression, by the 6 verse of the 14 Chap. Againe ỳ product of 14,348,907 by it selfe is 205,891,132,094,649, for the thirtieth and one bound of the progression by the 5 verse of the 14 Chapter. Now 3 — 1, and 205,891,132,094,649 — 1, are 2, and 205,891,132,094,648. Moreouer as 2 is to 1, so is 205,891,132,094,648, to 102,945,566,047,324. Therefore 205,891,132,094,649, + 102,945,566,047,324, that is, 308,836,698,141,973, is the desired summe of all the gifts receiued.

CHAP. XVI.

Of Disproportion.

1 THus farre of Proportion; Disproportion remaineth, which is when the qualities

liches of comparisons in quantity are diuerse.

Diuerse qualities are vnderstood whether the differences be diuerse, or reasons be diuerse. There is no great vse of this doctrine, and therefore may ye leaue it out: notwithstanding there are certaine of his rules commonly vsed, such as these are.

2 If two numbers be vnequall, the reason of the greater vnto the third shall be greater then the reason of the lesser vnto the same.

So in the assigned 5 3 and 2, the reason of 5 to 2, shall be greater then the reason of 3 to 2. Wherefore of two vnequall numbers that which shall haue the greater reason vnto the third, shall be greater. Againe

3 If the same number shall be compared with two vnequall, the reason of the compared to the lesser, shall be greater, then the reason of the same to the greater.

So if 5 shall be compared with 2 and 3, the reason of 5 to 2 shall be greater then the reason of 5 to 3. Wherefore if the same number shall be compared with two vnequall numbers, that to which ỹ compared shall haue the greater reason, shall be the lesser.

The end of the second booke.

THE ART OF ALLIGATION.

CHAP. I.

Of the definition of Alligation, and of his propertie.

1 ALLigation is an Art, which hauing certaine giuen, doth make the whole of the prices of ỹ mingled measures, equall to the whole of the prices of ỹ simple measures.

An 100 pound of siluer, of the price of 17, are to be mingled with another kind of siluer of the price of 24: so that ỹ whole of the prices of the weight of the mingled kinds of siluer, may be equall to the whole weight, of the prices of the weight of the simples. Here all ỹ prices are giuen: That Art therefore, which hauing these giuen, shall make such like sums equall, let it be Alligation.

Againe

Againe, of two kinds of corne mingled, let there be 10 bushels of the price of 16 ß a bushell, with 18 bushels of the price of 12 ß, foure bushels of ỳ mingled graine are so to be sold, that the whole of the prices of the mingled bushels 28, may be equall to the whole of the prices of the simple bushels 10 and 18. Here both the numbers & prices of the simple measures are giuen, and at the least wise some part of the number of the mingled measures. Therefore that Art which hauing such giuen shall make the assigned wholes equall, let it be Alligation.

2 The propertie of Alligation is, That in it the measures be like; but the price of the mingled measure is the meane in quantitie betweene the price of the simple measures.

A bushell of corne of 16 ß price, is to be mingled with a bushell of the price of 18 ß. Here by that propertie, first ỳ bushels are like measures. Then the price of the mingled bushell shall be the meane in quantitie betweene 16 and 18, that is to say, greater then 16, and lesser then 18. Therefore the price of the simple measure is called the Extreame or outmost, the price of the mingled the Meane.

CHAP.

CHAP. II.

Of the first Alligation and of his properties.

1 ALLigation is twofold: The first is that which crosseth the differences of the extreames from the meane.

It is manifest by the generall definition, that in euery Alligation certaine are giuen: Therfore in the first Alligation all the prices are generally vnderstood to be giuen. Furthermore here to alter or crosse the differences of the extreames, is to giue the difference of the least extreame to the greatest extreame, and contrariwise the difference of the greatest extreame, to the least extreame. This altering or crossing is therefore taken, because by it we do make the whole of the prices of the simple measures, equall to the whole of the prices of ỹ mingled measures. To the Declaration and Demonstration of this, two premises or rules are required. The first is thus: *If the same number shall multiply both certaine numbers and the summe of them, the product of the whole shall be equall to the whole of the products of the parts.*

As here: let the whole of 4 and 6 be 10, and let the number multiplying them all be 2: and then

then the products of 4 and 6 by 2, let them be 8 and 12, and the whole of them let it be 20. Therefore the product of 10 by 2, shall also be 20. The second premise or rule is thus: *If having three numbers unequall betweene themselues giuen, thou shalt multiply some one of them by the other, and shalt increase the same by one of the other, and shalt diminish it by the remaine, and then shalt multiply the increased by the subducted, & the diminished by the added; the whole of the latter products shall be equall to the whole of the former products.*

As here: let the three unequall numbers giuen be 4, 7, 9; Then multiply 9 by 4 and 7, and let the products be 36 and 63; and then the same 9 increase and diminish, as is aforesaid: to 9 therefore adde 7, and let the whole be 16: againe out of 9 take 4, and let the remaine be 5. Then multiply both 16 by 4, and 5 by 7: and let the products of them be 64 and 35, here the whole of 64 and 35 shall be equall to the whole of 36 and 63.

Those being thus determined, of two kinds of wine let one be of the price of 14 d, the other of the price of a 11 d: the difference from the meane 12, let them be 2 and 1. The whole therefore of them let it be 3. Now the meane price is 12 d, and ỳ extreame prices are 14 d, and 11 d. Therefore multiply the meane 12

by

by the whole of the differences, that is by 3, thou shalt make 36, that is, 3 firkins of the price of 12 ℔, shall be worth 36 ℔. Therefore I say, one firkin of the price of 14 ℔, with 2 firkins of the price of 11 ℔, are worth 36 ℔: that is, I say the whole of the products of 1 and 2 by 14 and 11, is 36. For here 3 is the whole of 1 and 2 as is aforesaid. And therefore here by the first premise if thou shalt multiply 12 by 2 and 1, the whole of the products 24 and 12 shall be equall to the product of 12 by 3. But we haue said ỹ 12 multiplied by 3, doth make 36: wherefore the whole of the products of 12 by 2 and 1 shall be 36.

Now these three, that is to say, 11, 12, and 14, are vnequall numbers betweene themselues: And the differences of the outmost 14 and 11, from the meane 12, are as we haue said, 2 and 1. Therefore the greatest extreame or outmost 14 is 12 + 2, and the least outmost 11 is 12 — 1. Therefore by the second rule, If thou shalt multiply the encreased 12 by the subducted 1, and the diminished 12, by the added 2, that is, if you shall multiply 14 by 1, and 11 by 2, the whole of their products shall be equall to the products of 12 by 2 and 1. But the whole of the products of 12 by 1 and 2, is 36, as it appeared. Therefore the whole

whole of the products of 1 and 2, by 14 and 11, shall be 26: which was to be declared. The cause therefore of the assumed altering of the differences is thus. *But in this place this is to be obserued: if both the numbers of the outmost be a number of many, the differences may be diuersly altered: And therefore euen in the same example there may be a diuerse Alligation made.*

Yet euery Alligation doth make the whole of the prices of the mingled measures, equal to the prices of the simple measures, as it appeareth by the definition of the generall Alligation: wherefore for the proposed equation, where both of the numbers of the extreames or outmosts shall be a number of many, there the altering of the differences may be at your choyce.

Now do follow two properties of the first Alligation, and as many kinds of the same. The first propertie is thus:

2 If the extreames or outmosts be equall in number, each of them is onely once compared with the meane.

As here

	18	2		20	3
				16	1
14			15		
	12	4		14	1
				12	5

The

The second property is thus:

3 If the extreames be vnequall in number, then euery extreame of the greater number, is onely once compared with the meane: but the extreame of the lesser number, if it be but one, then looke what the greater number of the extreames is, so oft the extreame of the lesser number is compared with the meane.

As here 18 3
14 3
11
8 7 | 3 | 10

In the example propounded, the greater number of the extreames or outmosts is two: and therefore the outmost of the lesser number is compared twise with the meane: where you shall obserue: *That the whole of many differences attributed to the same outmost, are taken for one difference.* For 7 and 3 we take 10.

4 But if there be many outmosts of the lesser number, if thou shalt compare more then one of them often with the meane, that number of comparison is not within the compasse of Art: Yet if but one, then looke how great the difference shall be of the vnequall numbers added to an vnitie, so oft it shall be compared with the meane: the other shall be but once onely compared with the same.

As

As here

	4	17				
	5	17				
	6	17				
	7	13				
15						
	28	8				
	32	11	10	9	30	

In this example the greater number of the outmosts is 4, the lesser 2; the difference of these unequall numbers is 2. to which if you adde 1, the whole shall be 3. Now I compare at ỹ least one of the extreames or outmosts of the lesser number, that is to say, 32, often with ỹ meane 15. Therefore that I compare thrice, the other but once onely with the meane. Lastly, also here I take the summe of the differences giuen to the same outmost, for one difference as before.

Thus farre of the two properties of the first Alligation; so many kinds of it do follow. To the vnderstanding of which the desired numbers are vnderstood to be the numbers of the simple measures.

CHAP. III.

Of the first Alligation of the first kind.

1 THE first Alligation of the first kind is, where hauing some part of the desired numbers giuen, we do conclude the other part by a simple proportion.

Here we take the difference attributed to that outmost, whose measures are giuen in some certaine number, for the first bound of the proportion: For the third we take the assigned number of measures. What this is, it shal be better vnderstood by examples following.

A farmer doth mingle 12 bushels of winter wheate, of the price of 14, with other graine wheate, barley & oates: the bushell of wheate is worth 18 d, of barley 11 d, of oates 9 d: a bushell of the mingled corne is bought for 10 d. Therefore how much wheate was there mingled? how much barley? and how much oates?

Here the assigned part of the desired numbers, is 12. Therefore first I alter the differences of the outmosts as you see.

	18	1
	14	1
	11	1
10		
	9	8, 4, 1
		13

Then

Then for the first bound of the proportion I take the difference giuen to that outmost, whose measures in number are giuen, that is to say, 1 bushell of the price of 14: for the third bound I take ỳ assigned number of measures, that is, 12 bushels of the price of 14. Therefore I conclude the propounded question by two simple proportions.

The first is thus:

1 Bushell of the price of 14, requireth 13 bushels of the prices of 9: therefore 12 bushels of the prices of 14, do require 156 bushels of the price of 9.

The second proportion is thus:

1 Bushell of the prices of 14, requireth 1 bushell of the other kinds: therefore 12 bushels of the price of the 14, do require 12 bushels of each other seuerall kinds of graine.

Wherefore here with 12 bushels of the price of 14, there were mingled 156 bushels of the price of 9; and 12 bushels of each seuerall kind of corne of the other sorts of graine.

Let there be 4 sorts of siluer: a pound of the first let it be worth 20, of the second 16, of the third 14, of the fourth 12: these foure sorts of siluer are so to be mingled that a pound of the mingled siluer may be worth 15. And there are taken 33 pound of the price of 16: how

many pounds are to be taken of ỹ other kinds?

Here the part of the deliuered numbers giuen is 33. Therefore the differences altered as you see.

20	3
16	1
15	
14	1
12	5

For the first bound of the proportion I take the difference attributed to ỹ outmost, whose measures in number are giuen, that is to say, 1 pound of the prices of 16: for the third bound I take the alligned number of measures, that is to say, 33 pound of the price of 16. Therefore I conclude the propounded question by three simple proportions.

The first is thus:

1 Pound, of the price of 16, doth require 1 pound, of ỹ price of 14: therefore 33 pound of the price of 16, do require 33 pound of the price of 14.

The second is thus:

1 Pound of the price of 16, doth require 3 pound of the price of 20: therefore 33 pound of the price of 16 do require 99 pound of the price of 20.

The third is thus:

1 Pound

1 Pound of the price of 16. doth require 5 pound of the price of 12: therefore 33 pound of the price of 16, do require 165 pound of the price of 12.

Taking therefore 33 pound of the price of 16, there may be taken 33 pound of the price of 14, 99 pound of the price of 20, and 165 of the price of 12. There may be taken I say, not that they are taken: for because that here both the numbers of the outmosts is a number of many: therefore the Alligation may be diuerse, as we warned before, at the definition of the first Alligation; keeping the same prices of the same kinds, if there had bene taken 6 pound of the price of 16, and 8 pound of the price of 12. so that a pound of the mingled may be worth 15, then thou shalt reason thus.

1 Pound of the price of 16, requireth 5 pound, of the price of 12: therefore 6 pound, of the of price 16, shall require 30 pound, of the price of 12.

Therefore to those 8 pound of the price of 12, there shall be added 22 pound of the same price. The other are most easie by the foresaid.

CHAP. IIII.

Of the first Alligation of the second kind.

1 THE first Alligation of the first kind is thus farre intreated of. The first Alligation of the second kind, is that which hauing the whole of the desired numbers giuen, doth conclude all the particulars by a proportion compounded by addition.

This compounded proportion is commonly called the Rule of fellowship. In it we take for the first bound of the proportion, the whole of the differences added: for the second the whole of the numbers of the measures: for the former bounds of the other reasons, the altered differences of the extreames from the meane.

A Vintner hath two sorts of wine, the one of the price of 6. the other of the price of 12: Out of these he desireth to draw 6 firkins of the price of 10: therefore how many firkins shall he draw of each sort?

Here the whole of the numbers of the simple measures giuen, is 6. Therefore the differences altered or crossed are as you see.

6 2

10

12 4

I take 6, the whole of the differences, for the first bound: for the second 6, and for the former bounds of the other reasons I take 2 and 4: and then I make the compounded proportion thus.

6 do yéeld 6, therefore	2	2
	4	4

Therefore here 2 firkins of the price of 6, are here to be drawne with 4 firkins of the price of 12. Hereupon it appeareth, that *if the whole of the numbers of the differences, be equall to the whole of the numbers of the simple measures, the numbers of the differences are the numbers of the simple measures.*

Hiero King of Syracusa in Sicily dedicateth a crowne of gold to the gods for his good successe. Let vs therefore suppose Hiero to haue giuen to the goldsmith for to make this crowne, 500 pound of gold: and the goldsmith to haue made a crowne of the iust weight, but in it to haue mingled some siluer. The King doth enquire of Archimedes how much there is of gold in the crowne mingled, and how much of the siluer?

For the answer of this doubt Archimedes toke two peces or wedges of mettall, and those of the same weight with the crowne, and of the same kind of mettall, of which the gold

and siluer were of the said crowne. Of this kind I say, (for let the wedges of siluer be of equall weight betweene themselues, but of vnequall purity) the purer wedge shall occupie lesse space, then the other shall do which is not so pure. The same is to be said of the wedge of gold. Therefore from hence there will spring a manifest error, except the mettals both the gold and siluer, shall be of the same kind. Therefore I say, both the wedge of gold and the wedge of siluer, were of the same kind of which the gold & siluer of the crowne were. Now let there be three such solid bodies taken. Let the first be the wedge of siluer, the second the wedge of gold, the third the crowne mingled of gold and siluer. And againe, let euery one of them be weighed in the same vessell full of water, and let the weight of the first be 968, of the second 952, of the third 964. And then let the weights themselues be taken for the prices of their bodies: therefore the price of the wedge of gold shall be 968. of the siluer 952, of the crowne 964. And now in the mingled crowne there are 500 pounds by the grant. Therefore here the whole of the numbers of the measure shall be 500. Therefore the differences altered as you see.

968

968 12:

964

952 4

I take the whole of the differences, that is to say, 16. For the first bound of the propor-tion: for the second the whole of the desired numbers of measures, that is, 500: for the for-mer bounds of the other reasons I take 12 and 4, and then I conclude the question by a compounded proportion.

4 125

16 500 Therefore

12 375

Whereupon it is gathered, that 125 pound of siluer was mingled in the crowne with 375 pound of gold.

A vintner hath 4 sorts of wine, of which he mingled 300 quarts: the quart of ỹ first wine is worth 12, of the second 10, of the third 9, and of the fourth 7. He selleth a quart of the mingled wine for 11: how many quarts there-fore did he mingle of the first, how many of the second, how many of the other kinds?

Here the assigned whole of the numbers of the simple measures is 300. Therefore the differences altered as you see.

	7	1			
	9	1			
	10	1			
11					
	12	4	2	1	7

I take the whole of the differences, that is, 10 for the first bound: for the second 300: for the formers of the other reasons I take 1, 1, 1, and 7; and then I make the compounded proportion thus:

		7	210	
10	300. therefore	1	30	
		1	30	
		1	30	

Whereupon I conclude that here there were mingled first 210 quarts of the price of 12, then 30 of each of the other kinds.

An Apothecary is to mingle 500 ounces of pepper, sugar, cinnamon and ginger. The ounce of pepper is worth 25, of sugar 24, of cinnamon 22, of ginger 18. Lastly, the ounce of the mingled spice shall be worth 23. How many ounces of pepper, how many of sugar, how many of the other kinds are here to be mingled?

Here first the assigned whole of the numbers of the simple measures is 500. Therefore the differences altered are as you see.

25	5
24	1
23	
22	1
18	2

I take the whole of the differences, that is to say, 9, for the first bound of the proportion; for the second 500: for the former bounds of the other reasons, I take 5, 1, 1, 2, and then I make the compounded proportion thus:

		5	$277\frac{7}{9}$
9	500 therefore	1	$55\frac{5}{9}$
		1	$55\frac{5}{9}$
		2	$111\frac{1}{9}$

Wherefore in this place there may be mingled $277\frac{7}{9}$ of pepper, of sugar $55\frac{5}{9}$ ounces, of cinnamon as many, of ginger 111 and $\frac{1}{9}$ of an ounce. There may be mingled, I say: not that there are so mingled. For here both the numbers of the outmosts, is a number of many, and therefore in this place the Alligation may be diuerse and manifold; and such a like manifold Alligation we obserued afore at the 3 Chapter, in the first Alligation of the first kind.

CHAP.

CHAP. V.

Of the second Alligation.

1 THus farre of the first Alligation: the second followeth which concludeth the meane by a proportion compounded by multiplication and addition.

Here both the numbers and prices of the simple measures are giuen, and at the least wise some part of their number being mingled. Therefore here we take the whole of the numbers of the simple measures, for the first bound of the proportion: the whole of the products of the same numbers by their outmosts, for the second. Lastly the assigned part of the mingled measures for the third bound of the proportion.

Of two kinds of graine there are mingled 10 bushels of the price of 16, with 18 bushels of the price of 12: what shall the price of the mingled bushell of graine be?

Here first I adde the numbers of the simple measures 10 and 18: the whole of the added is 28. Then I multiply 10 by 16, and 18 by 12, and I make 160, and 216, the whole of which is 376. The whole therefore of the prices of the simple measures is 376. But Alligation doth make the whole of the prices of

the

the mingled measures equall to the whole of the prices of the simple measures, as it appeareth by his definition. Therefore the mingled bushels 28 shall be worth 376. The question I do thus conclude.

28 mingled bushels are worth 376, therefore 1 bushell of the mingled graine is worth $13\frac{3}{7}$.

Let there be mingled 6 ounces of cloues of the price of 36, and 8 ounces of cinnamon of the price of 16, with 4 ounces of pepper of the price of 15: Of what price shall 4 ounces of the mingled spice be?

Here the numbers of the simple measures are 6, 8, and 4, the whole of which is 18. The products of 36, 16, and 15, by 6, 8, and 4, are 216, 128, and 60. The whole of which is 404. Therefore I say thus:

18 ounces of mingled spices are worth 404: therefore 4 ounces of the same shall be worth $89\frac{7}{9}$.

Let the same prices of the simple spices be kept: and let there be mingled $\frac{1}{2}$ ounce of cloues, and $\frac{1}{3}$ of cinnamon, with $\frac{1}{4}$ ounce of pepper, what shall the ounce of the mingled spice be?

Here first let the vnlike measures $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ be reduced vnto like. For the generall property

of

of Alligation doth require that the measure
be like: therefore first by the Reduction
parts, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{3}{4}$ there shall be found $\frac{6}{12}$
and $\frac{3}{12}$.

Then that the Numeration may be the
fier for the parts found by the Reduction
whole numbers, thou shalt take 6, 4, an
3. The whole therefore of the numbers
the measures shall be 13. Then I multipl
36, 16, and 15, by 6, 4, and 3, and I make 21
64, and 45: the whole of which is 325. Lastl
thou shalt conclude the question thus:

13 ounces of the mingled spice are worth
325: therefore 1 ounce shall be worth 25.

The end of Alligation.

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